CHAPTER 1
GENERAL AND PARTICULAR PROBLEMS IN TEACHING MATHEMATICS

1.1 INTRODUCTION

During the past few years, in most countries there has been an attempt to change and renovate teaching. In accordance with the demands of the times, serious steps have been taken to modernize the teaching of mathematics, and not only with respect to the contents, but regarding the aims and forms of presentation. Because today research in this subject area has become so comprehensive, there is more knowledge at our disposal, at a faster rate, than its value can be estimated. Thus, sometimes provision is made for trying to cope with these changes in unaccountable ways.

Along with the great knowledge "explosion" of the past decades in almost every subject area, impressive progress has been made in mathematics teaching. This progress has allowed didacticians in this country to reassess the contents and forms of their teaching. This has resulted in a clear shift in emphasis from memorized knowledge and mechanistic manipulations. Forms of teaching and learning which initiate true insight and refined concept formation, clear perceiving, and greater proficiency now are planned.

Proficiency in the meaningless manipulation of numbers and symbols must make an important place for insight into the meaning and structure of basic rules, axioms, and formulas. Now, the aim is to deal insightfully with these number and symbol relations in new situations. This renovation is directed to drastically diminishing the earlier unfruitful memorization and imprinting of learning materials, contents, relations, and methods.

However, every mathematics teacher remains explicitly aware that a minimum amount of ready knowledge, and a level of proficiency in mathematical manipulations are indispensable and are the basis for each mathematical operation or problem solution. Pupils who have memorized mathematical theory without insight often are able to reproduce it. However, as soon as they are given the task of applying such a theory, e.g., to solving new problems, fruitful

transfer and real achievement occur only if the matter is acquired with insight. Real insight and refined understanding can be used only if the pupils themselves can discover the essentials, relationships, and rules.

The cultural demands of the past few decades on purposefully educating (upbringing) and teaching have increased tremendously regarding their breadth and depth, with the result that increasingly more is asked of a child, without being able to provide extra time. With such a deluge of learning material, in his/her planning of learning contents, a teacher must differentiate between what appears to be **essential** or **incidental, invariant,** or **changeable.**

Because of the problem, as stated, it appears that reducing and disclosing the essentials of a slice of reality, and designing an accountable lesson structure, within which actualizing such unlocking and self-discovery is possible, can possibly contribute to the pupil attaining real insight and forming clear concepts.

The concept "reduction of learning material", in general, and "reduction of mathematics learning material", particularly, as well as their place and meaning in designing an accountable lesson structure, are the focus of this study. Before truly realizing the problems of teaching mathematics when real insight and clear concept formation are striven for, it is necessary to delimit and formulate these problems more clearly. In distinguishing between general and specific problems, there is an ordering by which specific ways of teaching and learning become relevant to making the above-mentioned aims possible.

1.2 GENERAL PROBLEMS

In the past, in secondary schools, great emphasis iss placed on commanding a large amount of factual knowledge and on mechanistically applying methods. Evaluation in such classrooms, as well as on common examinations, is focused too much on testing for readiness. Along with the renovation and expansion, which occurred in the past few years in the separate fields of science and in the various specializations of mathematics, a deluge of learning material became an actual problem and, on the other hand, a striving for completeness no longer is possible. Any striving for completeness occurs at the cost of originality. The result is that the pupils of mathematics are forced, because of their superficial

knowledge, to adopt "thinking patterns" and, for solving problems, to firmly grasp practiced recipe methods and algorithms.

Hence, today, there is a search for ways and forms of teaching and learning which allow for insight and clear concept forming, greater mobility and suppleness in disclosing and applying methods of solution to attain their rightful place in the future. The emphasis **shifts from** memorizing a great deal of ready quantitative knowledge, and the imitation of algorithms, **to** greater insight into the essentials of the concepts and structures of such an area of reality. That is, a teacher must remain directed to the fundamental concepts, relationships, and general methods in each subject area which leads itself to the greatest learning effect. By focusing on the most purposeful use of the different contents of the matter, a teacher can design problem situations in which a child can participate by perceiving, thinking, and remembering.

Regarding this, J. J. Dreyer(1) makes the following important point: "In the past, pupils have been taught **how** to perform a mathematical operation. For this purpose, they were given a 'rule', and emphasis was directed towards the achievement of results by strict observance of the 'rule', whether they understood what reason was behind it. The Modern Approach stresses the fundamental importance of an **understanding of concepts**. **Why** is now more important than **How.**"

By performing a reduction, a teacher discloses the essentials of the concept or subject to him/herself, and then incorporates them into his/her lesson aim. In planning a lesson structure, it is now necessary to shape the learning aim into a problem for a child. By further designing the lesson situation in such a way that there is a meaningful linking up with a child's foreknowledge, a teacher ensures, to a greater degree, that the pupils themselves **can** and **will** search for a solution. Therefore, here the emphasis is more on insightful learning, and less on learning devices and achieving apparent results.

A real perspective on the aims, which today are held for mathematics teaching, are only acquired after the above-mentioned course of learning is looked at. The compilers of a common, basic

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[1] Dreyer, J.J.,*Modern mathematics for South African Schools. Standard 7 (Grade 9)*. Preface.

syllabus for mathematics require that the following general aims be pursued. (These general aims are quoted as they appear in the appendix to **Spectrum, 42,** July 1966).

(i) To acquaint the pupils with the mathematical world within which everyday persons must measure and count;
(ii) to make pupils proficient in the use of mathematical knowledge and methods for solving questions (problems) and be able to find their way in various branches of vocational life;
(iii) to cultivate in pupils correct, accurate, and systematic work methods;
(iv) to give the pupils a basic insight, grasp, and knowledge of those basic mathematical principles which prepare them for further study in mathematics and in the natural sciences;
(v) to cultivate in the pupils a love for and interest in the further study of mathematics and the natural sciences.

From the above, it is seen that the emphasis has shifted from a mechanistic manipulation and thoughtless imitation to the acquisition of insight and to clear concept formation. Insight into the structure and unique nature of mathematics, as well as the mutually related and corresponding knowledge of its part-specialties, such as fruitful methods of solution, are an important part of this study. When insightful interpretations, forming clear concepts, integrating, and discovering fruitful methods of solution are the aims, only a method of teaching which involves the pupils in their totality has any chance of succeeding. Thus, a teacher has the task of planning and designing his/her lesson situation such that there are opportunities for original experiences, experiences of relevance, and independent discoveries of insightful learning, unambiguous interpretations, clear concept forming, and guiding to logical deductions.

To accomplish these aims, a teacher, in his/her preparation and planning of a specific lesson situation, must be able to distinguish clearly and unambiguously between what, as an essential of the contents, is omitted from his/her exposition of the new and which must be presented anew, and what is not essential, and must or can be left out of consideration. Such generally valid pronouncements and choices are only reached if a teacher him/herself has reduced the learning contents, and searched for what appears to be fundamental, necessary, and invariant. The creation of a fruitful moment for learning is only planned for scientifically if,

beforehand, a teacher has delimited the essentials of the topic for him/herself. This procedure, where the essence of the matter is searched for, is known in didactic theory as the "reduction of the learning material".

Now, to arrive at a systematic and ordered statement of the problem by considering the above views, there is a disclosure of some specific problems and arranging them in accordance with our aims for each phase of a lesson.

1.3 SPECIFIC PROBLEMS

1.3.1 Problems with the beginning of the lesson

To involve pupils in the learning event, the first requirement is that the design of the situation be meaningful to them. That is, a link must be sought between the new content and the pupils' experiential world [i.e., foreknowledge], and the learning material must be presented in such a way that they are enticed to participate. The problems of the beginning of the lesson are summarized in the following two categories: Problems related to linking up with foreknowledge, and questions regarding stating the problem.

(a) Problems with the lesson beginning

To meaningfully link up with the children's experiential world, it is necessary to consider the level of becoming and learning readiness of the class to be able to decide what foreknowledge must be actualized. Here, one distinguishes between problems with the child and problems with the learning material.

Problems with the child: When a child's potentialities and readiness are not considered, the beginning of the lesson can lead to general problems, e.g., errors the entire class is going to make, but also individual problems, because each child differs from the others. Thus, a teacher's introductory orientation can be at too high a level, or things can be assumed to be obvious, but which the pupils are not acquainted with. Where the beginning is on too easy a level, or is unnecessarily concrete, or confused, the interaction and harmony between the pupils' potentialities and readiness is disturbed, and this gives rise to boredom and frustration.

Problems with the learning material: (i) If, in his/her preparation, a teacher has not reduced the learning material to its essentials, he/she can confuse the child with his/her presentation because:

* He/she has not worked purposefully and, therefore, cannot help the pupils distinguish between details and what is essential to the matter;

* Therefore, he/she also is not able to select examples which already have sense and meaning for a child.

(ii) Since the foreknowledge and basic structure which are necessary for insight into the new concepts are not clearly recalled and made explicit, in first exposing the new contents, and later in its application, a teacher can discover that the pupils have deficiencies and uncertainties about the essentials of the matter. These resistances and deficiencies in a child's basic experiential world which a child undergoes so early while learning, can lead to an unnecessary waste of time, interruption, confusion, and stagnation.

(b) Problems with stating the problem

Lived experiencing the problem: A child's lived experience of the problem is differentiated in two ways: First, on a more pathic-affective level, as in the strangeness, unfamiliarity, and foreignness of the matter which, during sensing, as a mode of learning, is seen in a global understanding of the matter. About this, Sonnekus ([2]) says: "Sensing is a **totality act** of the person, or differently stated, a totality-in-communication with the world". On the other hand, as in mainly cognitive perceptions and disclosures, there is a search for the core and objective essentials of the object. In this reducing activity of perceiving, the problem is more clearly defined, and it becomes a problem-for-me (the learning person).

Giving meaning: Along with sensing and perceiving, there is an attempt to give meaning to what is viewed, by ordering and schematizing it. The matter first becomes a truly meaningful problem when a learner can relate it to his/her foreknowledge. After that, he/she realizes, however, that, what he/she knows about the matter has gaps and is an incomplete structure to be filled. Along with giving meaning, the first impetus for a child is to enter

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[2] Sonnekus, M.C.H., *Die leerwereld van die kind as beleweniswereld*, p. 63c.

the matter with the aim of finding a solution to the problem through attending to it.

Motivating: A teacher also has the task of presenting the learning aim such that its essentials show a formative quality and strongly appeal to and motivate a child, such that there is a clearly observable indication of greater attending, rapt focusing, and real interest.

The "New Seminar for Pedagogics" ([3]) at the University of Amsterdam states the following preconditions for stating the problem:

* "The problem must be in the pupils' sphere of interest".
* "All activity should lead the pupils to a solution".
* "The pupils must master the solutions they have found".

Own problem: The new contents should be presented in such a way that the problem is evident to a child because a teacher represents it so it can be interpreted more easily and formulated as one's own problem.

Searching for a solution him/herself: A teacher must present the problem such that it demands that the pupils themselves seek a solution, otherwise, they are forced, because of defective insight, to participate mechanistically in the problem-solving event. This can only lead to apparent results.

1.3.2 Problems (methods) with offering the new content

No methods can guarantee a fruitful lesson situation. Each teacher must try, in his/her presentation, to anticipate the most effective methods which lead to insightful learning. The following problems of offering the new content are distinguished: the problem of mechanical imitation, and the problem of premature generalization.

(a) The problem of mechanical imitation

In practice, we find that young and inexperienced teachers often blindly apply the methods they themselves have experienced in school. This so-called "recipe-like" form of teaching is not always

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[3] New Seminar for Pedagogics at the University of Amsterdam, Communication No. 74, p. 24b.

the most recent and fruitful of didactic views. We think here of the consistent use of the deductive method in teaching geometry, which for many years has been accepted for each lesson by teachers without reservation.

(b) The problem of premature generalization

Because more experienced teachers accept certain things as obvious, it can happen that there are premature generalizations, and a leading of pupils to rules and formulas long before they have arrived at insights into the essential characteristics or structure of the concept by experiencing varied situations, and adequate examples, e.g., in an inductive way. In this way, the learning person is forced to learn mechanically (by rote). Thus, there remains for him/her (a child) just one way out, i.e., to memorize the new concepts, rules, theorems, and formulas without acquiring insight into building up the structure, and into the meaning of the matter.

Van Rooy ([4]) directs the following warning against this: "Too often, it is forgotten that the contemporary form of our science is the end product of years of cooperative refinement by the most gifted thinkers."

1.3.3 Problems with controlling the new

The success obtained with programmed instruction, to a large degree, is that, as far as possible, the program is attuned to checking each act of thinking and unique achievement of each pupil. Although certain objections are raised against the behaviorist foundation of these aims, i.e., **immediate reinforcement** of the pupil, still it leads indirectly to the fact that each misunderstanding and error, incorrect interpretation, and blockage are immediately evident to the pupil. In this self-discovery of his/her potentialities, as well as his/her deficiencies lies the formative value of the instruction.

A child's checking is related to the act of guidance. At the beginning of each year, and with the presentation of each "new" topic, learning must be checked as often as possible. As the child acquires more insight, the rhythm of the checking can be decreased accordingly.

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[4] Van Rooy, D.J., *Die beginsels van die onderwys van Algebra en Rekenkunde*, p. 4b.

An experienced teacher anticipates which parts of the subject contain difficult relationships, abstract concepts, and refined methods of solution which are not easily understood by the pupils. Therefore, he/she selects good examples to illustrate the expected problems and possible stumbling blocks. An **inexperienced** teacher can only find out about these **essential** aspects through good preparation and, in this way, purposefully pause and give special attention to them.

1.3.4 Problems of application

In this phase of the lesson, usually the problem arises of what examples to use and how many. This problem is differentiated further, and is treated in more detail below.

(a) The choice of unimaginative and unrealistic examples which have no clear use or which appear so easy that they do not direct an appeal to the child.
(b) Excessive drill work and repetition, only for the sake of the repetition itself, give rise to frustration.
(c) Foreign to life and abstract assignments contribute to a child constructing two worlds, i.e., that of the school and that outside the school.
(d) It often happens that fundamental concepts and operations are learned in isolation without becoming integrated with the foreknowledge, as a larger functional unity.

1.3.5 Problems of evaluation

According to Van Hiele ([5]), there can only be an insightful activity when a pupil "purposefully acts adequately in new situations". Requirements meeting this precondition are summarized in three groups.

(a) By asking about definitions, formulas, theorems, and rules, one can test whether the pupils know their theory. However, this is not yet a test of insight and understanding.
(b) By asking questions about where the newly acquired theory must be applied to reach a solution, a teacher can determine the

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[5] Van Heile, P.M., *De problematiek van het inzicht*, p. 11b.

extent to which the pupils have insight into the essentials of the matter.
(c) Assignments also can be given in which a child must construct a new structure or relationship from his/her previous mastery.

A good test must always deal with all three aspects. A teacher can only delimit and point out the **core learning material** so that it is insightfully mastered by all pupils and made their own by good preparation and thoroughly reducing the contents. Only when pupils achieve on tests with respect to these contents can there be a move to supplemental and orienting programs.

1.4 SUMMARY

All the general problems regarding guidance to insightful interpretations, clearer concept formation, and the discovery of good methods of solution, as well as the problems at the beginning, the course, and the end of a lesson require careful reflection and scientific research before an accountable lesson structure can be constructed. Therefore, to be able to view the relationship of these problems within the totality of the lesson structure in the right perspective, and to be able to make meaningful comments, certain didactic findings are looked at more closely.

Solutions to the mentioned problems should be sought in didactic theory so that there is less need in future planning and designing of lesson situations for relying on chance. Thus, there should be a search in didactic theory for findings regarding "the reduction of learning material", and its place and value in the lesson structure. Therefore, it is appropriate to proceed in the second chapter to a discussion of the question of "the reduction of learning material", and the possibilities, and meanings this opens for practice, to the extent that it can contribute to helping solve the above **problems.**

However, before the essentials of the concept "reduction of learning material" can be penetrated, it is first important to say something about **learning material** and the selection of suitable **learning contents.**

There also is an attempt to answer the question of what reduction is. To be able to venture an answer, there is a search for connections with the phenomenological method, which essentially is a method of reduction. Therefore, the phenomenological method and views of

Husserl and others are considered with the aim of explaining and introducing some concepts.

Then, the concept reduction is put in its didactic context and elaborated as a didactic category, which is its relevance for practice. Of course, one cannot neglect indicating that a second order situation, such as a school, actualizes this course in reverse. Where, in general didactic theory, it is accepted that the form of teaching is universal, and the content specific, it is important to note that, in the planning of a lesson situation, the content, as indicated in the different curricula, are firmly in surrounding reality. Proceeding from the learning content, and its essentials, as learning aims, there then is a search for a lesson form which he/she will bring out the best quality of the content.

In the third chapter, there is an indication of the place and value of reducing the learning material in the different phases of the lesson structure.

Also, to be able to give sense and meaning to all these theoretical views and findings and allow them to have the most direct relevance for practice, we proceed to lesson preparation in mathematics in the last chapter.

In terms of topics from the mathematics syllabus, it is shown how the theory of the previous chapters can be applied to practice. Also, the examples can serve as **guidelines** for mathematics teachers who, in the future, are going to plan lesson situations themselves.