CHAPTER 4

# PRACTICAL EXAMPLES TO ILLUSTRATE THE SENSE AND MEANING OF REDUCING THE LEARNING MATERIAL FOR CONSTRUCTING A MATHEMATICS LESSON STRUCTURE

## 4.1 INTRODUCTION

In the previous chapters, a theoretical explication of the place and value of reducing the learning material for each phase of the course of a lesson is given. Now, to give such theoretical pronouncements content, in this chapter, several examples are presented which serve as guidelines by which a practice can be planned and implemented.

For the sake of simplifying the matter, and directly indicating relationships between the activity of reduction and the relevant aspects of the larger unity of the lesson, the explication in this chapter is divided into two main moments:

a) Several themes are chosen from the mathematics syllabus in terms of which an attempt is made to construct effective examples for each successive phase of the lesson structure. At first, to illustrate each phase of the course of the lesson, a separate theme is chosen and reduced to its essences. The aim is to show the relationship between the essences of the matter, as disclosed by the reduction step, and the successive sequence structures (phases) such as, e.g., actualizing foreknowledge, stating the problem, and others. This then serves as a guide for a teacher, and, at the same time, he/she has an opportunity to acquire step-wise experience with the problems which show up during the reduction activity.

b) Because these aspects of the lesson structure cannot be meaningfully understood separate and apart from each other, a summarized image, therefore, is given in which the entire course of a lesson is planned in terms of one theme. It is hoped that such a complete image of an example lesson succeeds in offering a total image of the interaction between lesson planning and reducing the learning material, and which must serve as a model for future designs.

## 4.2 AN EXAMPLE FOR EACH PHASE OF THE LESSON STRUCTURE

With the successive phases of the course of the lesson as the point of departure, some themes are given to show the meaning of reducing the learning material for these divisions of the lesson structure.

### 4.2.1 The learning aim and reducing the learning material

a) *Theme:* (Algebra, Standard 6 [Grade 8]).
Cardinal number (definition).

*Remark:* A cardinal number is viewed as the common unit of equivalent aggregates:

*Notation:* n {a; b; c} = 3.

b) *Reduction of the theme*
The essence of this theme is in:

(i) *The concept "cardinal number"*

It is a number that merely indicates the number of units of something, but not their order. For example, in the formulation "three apples", "3" is the cardinal number. The common units of the aggregates {a; b; c} and {1; 2; 3} is that both include the same "number" or number of elements; therefore

{a; b; c} $\downarrow$ {1; 2; 3}.

(ii) *The notation system*

{a; b; c} = 3.
Or, if A = {a; b; c} n(A) = 3.
n = cardinal number.

(iii) *The meaning of the cardinal number in the number system.*

{cardinal numbers} =
{natural numbers} from {0}.

c) *The learning aim*
The essence as manifested by the reduction of the theme is the learning aim which a child is going to make his/her own.

This culminates in the following aspects:
(i) Insight into the concept cardinal number.
(ii) The insightful mastery of the notation system, e.g.,
n {a; b; c} = 3.
(iii) The distinctive characteristics of the cardinal number with respect to other types of numbers such as natural numbers.

**4.2.2 The lesson aim and reducing the learning material**

a) *Theme* (Geometry, Standard 7 [Grade 9]).
If two lines intersect, the opposite angles are equal.

b) *Reduction of the theme*
When the teacher looks for this theme in the syllabus and searches for its essences, he/she can delimit as essential the concept *opposite,* as well as the *method* by which the proof is going to progress.

c) *The learning aim*
The insightful mastery of the new concept (opposite angles), as well as the specific solution method (proof of the statement) is a child's *learning aim.*

d) *The lesson aim*
The lesson aim is realized during the lesson. The essence of the matter, as made visible to a child in the learning aim. is the basis of the didactic design. The lesson aim, as it has acquired form in the didactic design, thus, embraces the anticipated form of the lesson as well as the planned didactic modalities.

To reach the essence of this proposition regarding opposite angles, and as seen in the reduction steps, the lesson aim must embrace the following:
**I. Unlocking the essential of the concept "opposite angles"**

**Theory** **Example**

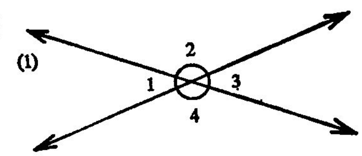
a) (i) *Meaningful unlocking of everyday language usage:* side, opposite side, upper side, opposite, opposite to, opposite each other, etc. (This involves spatial orientation and the location of one matter with respect to another.)

1. Shops that are opposite each other on both sides of the same street.
2. Opponents in the two opposite corners of a wrestling or boxing ring.
3. The opposite angles of rectangles and parallelograms are equal.

(ii) The concept "opposite" in geometric contexts.

(ii) *Example:* Pairs that cross such as lines that intersect.

(1)




(iii) *A formal description of the concept "opposite":*

(1) When two lines intersect each other, two pairs of opposite angles are formed at the point of intersection.

$\hat{O}_1$ and $\hat{O}_3$
$\hat{O}_2$ and $\hat{O}_4$

(2)The size of each pair of opposite angles can differ.

(2) $\hat{O}_1$ and $\hat{O}_3$ are acute angles.
$\hat{O}_2$ and $\hat{O}_4$ are obtuse angles.

(3) A pair of opposite angles always is equal.

(3) $\hat{O}_1 = \hat{O}_3$
$\hat{O}_2 = \hat{O}_4$

**Theory**

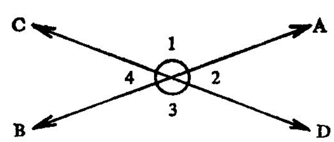
**Example**

b) Proof of the proposition:

*Illustration*

(i) Diagrammatic representation

(i)




*Proof*

(ii) Linking up with fore-knowledge re *straight angles.*

(ii) $\hat{O}_1 + \hat{O}_2 = 180^0$
$\hat{O}_2 + \hat{O}_3 = 180^0$

(iii) The "equality" concept.

(iii) $\hat{O}_1 + \hat{O}_2 = \hat{O}_2 + \hat{O}_3$

(iv) The concept "common angle".

(iv) But $\hat{O}_2$ is included on both sides.

$$\hat{O}_1 + (\hat{O}_2 - \hat{O}_2) =$$
$$\hat{O}_3 + (\hat{O}_2 - {}_2\hat{O})$$
$$\hat{O}_1 = \hat{O}_3$$

(v) *Control (check, verify)*

(v) Only thus can it be proven that $\hat{O}_2 = \hat{O}_4$.

**II. Lesson form**

a) *Ground form*

(i) *The concept "opposite angles"*
In terms of a *conversation* about the examples mentioned (shops, ring, pairs and lines) there is a meaningful linking up with the everyday language usage and a push through to the formal definition of the concept opposite angles in a geometric context.

(ii) *The proof*
For the exposition of the formal proof of the proposition the same ground forms are chosen again, namely conversation and example.

b) *Methodological principles*

(i) *Elucidating the concept: Inductive*
To be able to arrive at the formal definition of the concept opposite we begin with everyday examples and work through to the general definition of proposition.

(ii) *The proof: deductive*

In proving the proposition we are going to work deductively where from the formal description of the proposition there is a push through to particular examples (problems).

c) *Principle of ordering: Punctual*
Where the proposition is taken as the point of departure for the presentation this always is an indication that a punctual ordering can be successful.

Here one goes from the difficult and complex proposition in search of a suitable example that can illustrate further its various characteristics. Each time there is a return to the formulation of the proposition as the point of departure. We provide a brief example:

(i) *General postulate:* If two lines intersect each other the opposite angles are equal.
(ii) *First characteristic:* The concept opposite
(iii) *Second characteristic:* Two lines intersect each other.
(iv) *Third characteristic:* Equal angles.
(v) *Fourth characteristic:* Relationship among the first three characteristics.

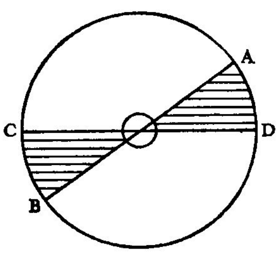
## III. Didactic Modalities

a) *Principle of actualization* — *Guided activity*

b) *Modes of learning*

(i) *Sensing.* By making use of a cardboard model where the size of the angle can be changed and a particular color effect shown, a pupil can acquire a first, global idea of the matter.

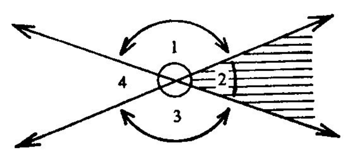
(i)




Line AB moves around O.

(ii) *Perceiving.* The totality view now must be broken open and the problematic isolated.

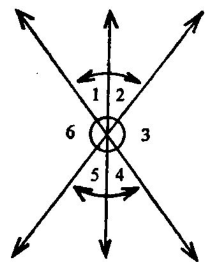
(ii)



Because $\hat{O}_1 + \boxed{\hat{O}_2} = 180^0$
and $\boxed{\hat{O}_2} + \hat{O}_3 = 180^0$
$\therefore$ $\hat{O}_2$ is common.

(iii) *Thinking.* Allow the child to point out opposite angles in a more complex figure.

(iii)




For example: $\hat{O}_1 + \hat{O}_2 = \hat{O}_4 + \hat{O}_5$.

(iv) *Remembering.* 1. A good example is worked through with the pupils to control their insights into the new.
2. A number of examples are worked through by the pupils themselves.

(iv) *Guided functionalizing*

Thus the lesson aim includes much more than the learning aim in the sense that the teacher for his presentation must anticipate and actualize certain supporting and supplementary principles, forms and modes of learning.

### 4.2.3 Actualizing foreknowledge and reducing the learning material

a) *Theme:* (Algebra, Grades 11 and 12).
Proof of the remainder theorem.

*b) Reduction of the theme:*

The essence this theme is that When a polynomial in x is divided by $(x - p)$ then the remainder is the same polynomial in p.

**Illustration**

Suppose the polynomial $t(x)$ is divided by $(x - p)$. When we now accept that the quotient is going to be $k(x)$ and the remainder R (where R is zero) this means the following:

$t(x) \equiv (x - p \text{ x } k(x) + R$

Let $x = p$.

$\therefore t(p) = (p - p) \text{ x } k(x) + R.$
$= O \text{ x } k(x) + R$
$= O + R.$

*Disclosure:* The remainder immediately can be calculated by entering the polynomial in p.

c) *Actualizing foreknowledge*
The following foreknowledge as a supportive basis must be actualized beforehand.

| **Theory** | **Illustration** |
|---|---|
| (i) *The notation system.* | (i) $t(x) = x^2 - 4$<br>$t(2) = (2)^2 - 4$<br>$= 4 - 4$<br>$= 0.$ |
| (ii) The division as carried out with a polynomial as dividend and another polynomial of a lower value as divisor. Also the quotient must not be negative and there must be a remainder. | (ii) $x + 2$<br>$x - 5 \;\overline{)\, x^2 - 3x + 5}$<br>$\underline{x^2 - 5x}$<br>$2x + 5$<br>$\underline{2x - 10}$<br>$+ 15$ |

$\therefore x^2 - 3x + 5 \equiv (x - 5)(x + 2) + 15$

Dividend = divisor x quotient + Remainder.

### 4.2.4 Stating and formulating the problem and reducing the learning material

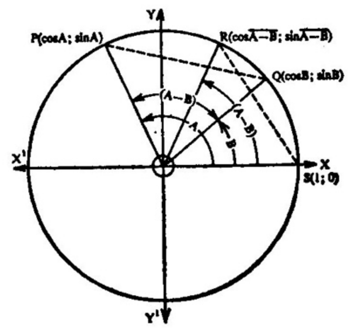
a) *Theme* (Trigonometry, Grades 11 and 12).
Compound angles. Proof of the formulas cos $(A \pm B)$ and Sin $(A \pm B)$.

b) *Reduction of the theme*

| Theory | Illustration |
|---|---|
| (i) The essence of this theme can be carried back to the proof of the formula *cos (A + B) = cos A cos B – sin A sin B.* The formulas for cos (A + B) and Sin (A B) all are derived from the above through substitution. | cos (A – B) = Cos A cos B + sin A sin B.<br>(i) Substitute B with –B.<br>$\therefore$ cos [A – (–B)] = cos A cos (–B) + sin A sin (–B)<br>***cos (A+B) = cos A cos B – sin A sin B.*** |
| (ii) The essence of the proof of cos (A – B) = cos A cos B + sin A sin B is in implementing the interval formula $(AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ between two points $A(x_1; y_1)$ and $B(x_2; y_2)$ as defined in analytic geometry. | (ii) Substitute A by $(90^0 - A)$.<br>$\cos(90^0 - A - B) = \cos(90^0 - A) \cos B + \sin(90^0 - A) \sin B.$<br>$\cos [90^0 - \overline{A + B}] = \sin A \cos B + \cos A \sin B$<br>sin (A + B) = sin A cos B + cos A sin B.<br>(iii) Substitue B by –B in the formula sin (A + B) |

sin (A – B) = sin A cos (-B) + cos A sin (-B).

sin (A – B) = sin A cos B – cos A sin B.




PQ = RS (center angles Equal)

$PQ_2 = RS_2$

$\therefore (\cos B - \cos A)^2 + (\sin B - \sin A)^2 = (\cos \overline{A-B} - 1)^2 + (\sin \overline{A-B} - 0)^2$

$\therefore \cos^2 B - 2 \cos A \cos B + \cos^2 A + \sin^2 B - 2 \sin A \sin B + \sin^2 A = \cos^2 \overline{A-B} - 2 \cos \overline{A-B} + 1 + \sin^2 \overline{A-B}$

$\therefore (\sin^2 B + \cos^2 B) + (\sin^2 A + \cos^2 A) - 2 \cos A \cos B - 2 \sin A \sin B = (\cos^2 \overline{A-B} + \sin^2 \overline{A-B}) - 2 \cos \overline{A-B} + 1$

$\therefore 1 + 1 - 2 \cos A \cos B - 2 \sin A \sin B = 1 - 2 \cos \overline{A-B} + 1$

$\therefore 2 - 2(\cos A \cos B + \sin A \sin B) = 2 - 2 \cos \overline{A-B}$

$\therefore 2 \cos(A - B) = 2(\cos A \cos B + \sin A \sin B)$

$\therefore \cos(A-B) = \cos A \cos B + \sin A \sin B.$