c) *Stating and formulating the problem*

(i) In terms of the foreknowledge actualized the pupil can realize that cos (A – B) is *not* equal to cosA – cosB but is equal to cosA cosB + sinA sinB. The question that arises is: How can this special case be shown to be generally valid?

Illustration

(i) If $\hat{A} = 60^0$ and $B = 30^0$,
$\hat{A} - B = 30^0$.

$\therefore \cos(A - B) = \cos 30^0$
$= \frac{\sqrt{3}}{2}$

$\cos A - \cos B = \cos 60° - \cos 30°$
$= \frac{1}{2} - \frac{3}{2}$
$= \frac{1 + \sqrt{3}}{2}$

$\cos A \cos B + \sin A \sin B$
$= \cos 60° \cos 30° + \sin 60° \sin 30°$
$= \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$
$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$
$= \frac{2\sqrt{3}}{4}$
$= \frac{\sqrt{3}}{2}$

The problem culminates in the pupils knowing:

(1) Where the solution cosA cosB + sinA sinB comes from. In other words how can it be demonstrated that cos (A – B) always is equal to cos A cosB + sinA sinB.

(2) The pupils are going to search alone for possible solutions to the other cases.

4.2.5 Exposing the new and reducing the learning material

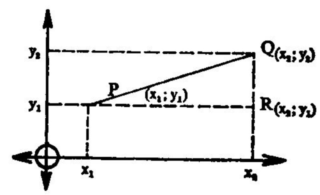
a)*Theme:* (Analytic Geometry, Grades 11 and 12).
The length of a line connecting two particular points.

b) *Reduction of the theme*
The essence of this theme is disclosed in the agreement shown with the Pythagorean theorem.

Theory

The essence is that the distance between any two points on a Cartesian plane can be determined by computing the hypotenuse when the right angle sides are known.

Illustration

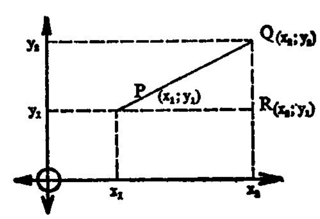


$PR = x_2 - x_1$
$RQ = y_2 - y_1$
$PQ^2 = PR^2 + RQ^2$ (Pythagoras)
$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

c) *Exposing the new*
The exposition of the new exists in guiding the pupils to the new structure (length of PQ) by applying the Pythagorean theorem.

Exposition

Illustration



(i) On the basis of foreknowledge regarding the graphic:

(i) $PR = x_2 - x_1$
$QR = y_2 - y_1$
$PQ = ?$

(ii) The application of the Pythagorean theorem.

(ii) $PQ^2 = PR^2 + QR^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

(iii) Square root.

(ii) $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

4.2.6 Control of the new and reducing the learning content

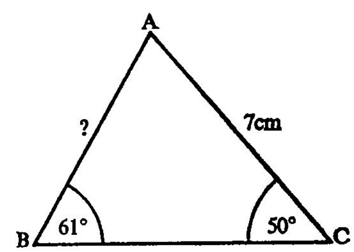
a) *Theme* Trigonometry Grades 11 and 12).
Application of the sine rule for solving acute and obtuse triangles.

b) *Reduction of the theme*
The value of this formula is that the sine rule is used:

(1) When the size of two angles and the side opposite one of them is given. (When a side is given that is not opposite one of the two given angles, the size of the required angle always first be computed).

Theory	Illustration

(i) Given: Two angles and a side opposite one of them.



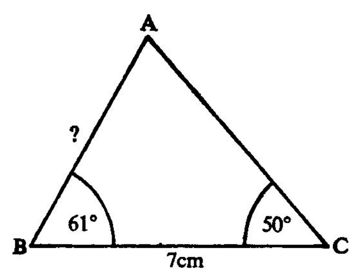
$$\frac{AB}{\sin 50^0} = \frac{7}{\sin 61^0}$$

$$\therefore\ AB = \frac{7 \sin 50^0}{\sin 61^0}$$

(ii) Two angles and a side that is not opposite one of the given angles.

Compute $\hat{A}$.

(ii)



$$\hat{A} = 180^0 - (61^0 + 50^0)$$
$$= 180^0 - 111^0$$
$$= 69^0$$

Now compute AB.

$$\frac{AB}{\sin 50^0} = \frac{7}{\sin 69^0}$$

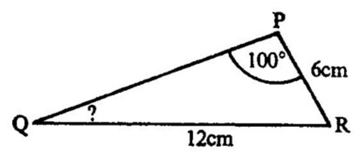
$$\therefore AB = \frac{7 \sin 50^0}{\sin 69^0}$$

(2) When the length of two sides and the angle opposite one of them is given.

Theory

Two sides and an angle opposite one of them.

Illustration



$$\frac{\sin Q}{6} = \frac{\sin 100^0}{12}$$

$$\sin Q = \frac{6 \sin 100^0}{12}$$

c) *Control of the new*

Now that an example where two angles and a side opposite one of them is given and demonstrated by the teacher the following example can be done together to control or verify if the pupils have arrived at insight.

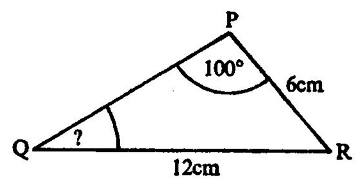
In triangle PQR:

PR = 6 cm, QR = 12 cm and QPR = 100^0.

As an example for actualizing the new insights an obtuse triangle is chosen in order to review anew with the pupils the application of the rule. In this example we begin with the notation

$$\frac{\sin Q}{6} = \frac{\sin 100^0}{12}$$

that will assist in loosening the pupil from the original notation $\frac{\sin A}{a} = \frac{\sin B}{b}$. At this stage it also can be expected of the pupils that they already must show mobility in the use of the rule always beginning with the *unknown* on the left side of the formula.

Theory	**Illustration**
	
(i) Write the formula	$\frac{\quad}{6} = \frac{\sin 100^0}{12}$
(ii) Solve for Q	$\sin Q = \frac{6\sin(180^0 - 80^0)}{12}$
	$= \frac{6 \sin 100^0}{12}$

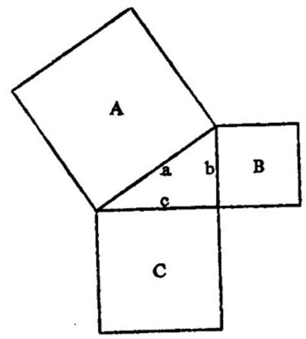
4.2.7 Functionalizing and reducing the learning material

a) *Theme:* (Geometry, Grades 11 and 12).
The Pythagorean theorem.

b) *Reduction of the theme*
The essence of this theme is that the square of the hypotenuse of a right triangle is equal to the sum of the square of the sides forming the right angle.

Theory | **Illustration**

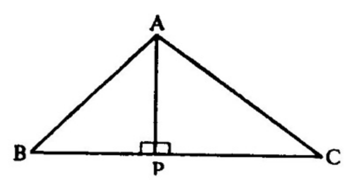
The sum of the area the squares drawn on the shorter sides of a right triangle are equal to the area of the square drawn on the longest side.



$A = B + C$;
$a^2 = b^2 + c^2$

c) *Functionalizing the new insights*
For a practicing *of* insights the following example can be carried out by the pupils themselves.
In triangle ABC, $\hat{A} = 90^0$ and AP > BC. Prove that $AB^2 - AC^2 = BP^2 - PC^2$. With this example the application of the Pythagorean theorem is carried out.

Theory | **Illustration**



Proof:

(i) Following the Pythagorean theorem. — (i) $AB^2 = AP^2 + BP^2$

(ii) Following the Pythagorean theorem. — (ii) $\underline{AC^2 = AP^2 + PC^2}$
$AB^2 - AC^2 = BP^2 - PC^2$

The newly acquired geometrical insights can be integrated with already existing insights in Graphics, Trigonometry, Analytic Geometry and Arithmetic in terms of the following examples:

(i) Graphics.	(i) Draw the graph of $x^2 + y^2 = 15$.
(ii) Trigonometry.	(ii) In triangle PQR, Q = 90^0. Prove that $\sin^2R+\cos^2R = 1$.
(iii) Analytic Geometry.	(iii) A is the point $(x_1; y_1)$ and B is the point $(x_2; y_2)$. Derive a formula to indicate the length of AB.
(iv) Arithmetic.	(iv) A 1 0.5 meter wire is affixed to a point at the top of a telegraph pole and is anchored in the ground 3.6 meters from the pole. Calculate the length of the pole.

4.2.8 Evaluating and reducing the learning material

a) *Theme:* (Trigonometry, Grades 11 and 12).
Simple identities that pertain to the relationships mentioned in the above section.

b) *Reduction of the theme*
The essence of this theme is: To simplify the left and the right sides of the identity to the same expression.

Theory	**Illustration**
Prove that $\tan A+\cot A \equiv \mathrm{Cosec}^2A.\sec A.\sin A$.	
(i) *Left side:* Simplify to a sin- and cos-function.	(i) Left side

$$= \tan A + \cot A$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\cos A.\sin A.}$$

$$= \frac{1}{\cos A.\sin A.}$$

(ii) *Right side:* Simplify to a sin- and cos-function.

(ii) Right side

$$= \operatorname{cosec}^2 A.\sec A.\sin A.$$

$$\frac{1}{\sin^2 A} \times \frac{1}{\cos A} \times \sin A$$

$$= \frac{1}{\sin A.\cos A.}$$

(iii) The solution now must necessarily fall open.

$\therefore \tan A + \cot A = \operatorname{cosec}^2 A.\sec A.\sin A.$

c) *Evaluating*

The following examples can be given to the pupils on a test:

(i) For the proof of the identity: $\tan A + \cot A \quad \operatorname{cosec}^2 A \sec A \ \sin A$, the pupils alone must fill in the open spaces of the following exposition.

Left side = $\tan A + \cot A$.

= ....

$$= \frac{\sin^2 A + \cos^2 A}{\cos A.\ \sin A.}$$

= ....

Right side = $\operatorname{cosec}^2 A.\sec A.\sin A$.

= ....

= ....

This question is posed, on the one hand, so as to evaluate the pupils' skillfulness and knowledge with applying the newly acquired insights. On the other hand, by this the possibility that the pupils are going to make errors with irrelevant manipulations is eliminated.

(ii) Prove that $\tan^2 \equiv \sin^2x + \sin^2x.\tan^2x$. With this example, on the one hand, there is an attempt to evaluate insight into the learning aim (implementing relationships). On the other hand, the mobility of the child regarding the application of his acquisitions in a new situation is tested.

Theory	**Illustration**
	Right side
(i) Common factor	$= \sin^2x+\sin^2x\tan^2x$ (i) $= \sin^2x(1+\tan^2x)$
(ii) Relationship: $1 + \tan^2x = \sec^2x$.	$= \sin^2x(\sec^2x)$
(iii) Relationship: $\sec^2x = \dfrac{1}{\cos^2x}$	$= \sin^2x \dfrac{1}{\cos^2x}$ $= \dfrac{\sin^2x}{\cos^2x}$
(iv) Relationship: $\tan^2x = \dfrac{\sin^2x}{\cos^2x}$	Left side $= \tan^2x$ $= \dfrac{\sin^2x}{\cos^2x}$
(v) Solution.	$\therefore \tan^2x = \sin^2x+\sin^2x.\tan^2x.$

4.2.9 Summary

Because the various facets of the lesson structure cannot exist meaningfully apart from each other a summary example is given within which the complete course of a lesson is planned in terms of a particular theme. Each step sequentially is brought into harmony with the insights acquired by reducing the learning material to its essences.

4.3 AN EXAMPLE OF A COMPLETE LESSON STRUCTURE

a) *Theme:* (Trigonometry, Grades 11 and 12).
Solving trigonometric equations. Solutions must be limited to the interval [- 360^0; 360^0].

b) *Reduction of the theme*
The essence of this theme is determining the collection of all *sets* of angles within the previously mentioned interval that satisfies the equation. This embraces the reversal of the formula by *simplifying* and/or *factoring* it into a form by which the solution can be found from the relevant tables.

Theory	**Illustration**
The course of the solution	$2 \cos x = 3 \tan x$.
(i) Substitution.	(i) $2 \cos x = 3 \sin x / \cos x$
(ii) Simplifying.	(ii) $2 \cos^2 = 3 \sin x$.
(iii) Substitution.	(iii) $2(1 - \sin^2 x) = 3 \sin x$.
	$2 - 2 \sin^2 x = 3 \sin x$.
	$-2\sin^2 x - 3\sin x + 2 = 0$
(iv) General form.	(iv) $2 \sin^2 x + 3 \sin x + 2 = 0$
(v) Factorizing.	(v) $(2\sin x - 1)(\sin x + 2) = 0$
(vi) Solving for the size of the unknow angles from the tables.	(vi) $2 \sin x - 1 =$ X $\sin x + 2 = 0$
	0
	$2 \sin x = 1$ $\sin x = -2$
	$\sin x = 1/2$ unsolvable
	$x = 30^0 + k360^0$
	or $x = 150^0 + k360^0$
	$x = 30^0$; 150^0
	-330^0; -210^0.

c) *The learning aim*
The essence of the theme as shown by the reduction activity of the teacher is summarized as follows:

(i) The effective implementation of known *identities* and *factors* by simplifying the equations to a *general form* by which the solutions can be found from the relevant tables.

(ii) The solution of the unknown size of the angles within the given interval that the equation satisfies.

d) *Lesson aim*

In compliance with the learning aim now the teacher proceeds to plan his lesson aim. That is, he anticipates a minimum amount of foreknowledge, an effective lesson form and effective modes of learning. The lesson aim only can be realized in the course of the lesson. Therefore, the lesson form and didactic modalities that are planned must be indicated for each phase of the lesson.

Actualizing foreknowledge

The course of a lesson only can have a meaningful beginning if room is made in the lesson aim for actualizing relevant foreknowledge. The following is a guide to what foreknowledge must be actualized in solving trigonometric equations.

(i) *Known identities*

(i)

$$\sin x = \frac{1}{\operatorname{cosec} x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

(ii) *Factors*

1. Trinomial.
2. Grouping.
3. Common factor.

(ii)

1. $2x^2+3x+2$.
2. $5xy-4x-15y+12$.
3. Compare 2.

(iii) *Quadratic equations.*

(iii) $2x^2+3x-2=0$

(iv) The use of the *natural tables.*

(iv) From the tables determine the size of x if sin x = 0.5.

The following lesson form and didactic modalities are anticipated for this phase of the lesson.

A. Lesson form

a) *Didactic ground form(s).*
Conversation and example.

b) *Methodological principle.*
Mainly inductive.

c) *Principle of ordering learning material.*
Linear.

B. Didactic modalities

a) *Principles of actualization*
Guided activity.
Guided individualization.

b) *Modes of learning.*
Remembering.
Reviewing and exercising
Known insights.

c) *Teaching aids.*
Schematizing on the blackboard.

Stating and formulating the problem

In terms of the relevant foreknowledge actualized the pupils are made aware that at this stage they can solve algebraic equations and even simple "trigonometric equations" such as sin x = 0.5.

However, the first totality view of an equation such as 2 cos x = 3 tan x now immediately awakens in the child a feeling of not knowing, of strangeness or something problematic. By a closer investigation (perceiving), i.e., by analyzing and "breaking open" the problem (trigonometric equations) certain things still remain unsolvable. The child with his foreknowledge of algebraic equations cannot independently arrive at a solution. With a good example the child now is helped to formulate the problem for himself:

The problem with respect to trigonometric equations can be summarized in the following two steps:

Example

(i) Defining the original equation as an equation with only one function.

(i) *Original equation.*
$2 \cos x = 3 \tan x.$
Changed equation.
$2\sin^2 x + 3\sin x - 2 = 0.$

(ii) Factoring the new equation.

(ii) $(2\sin x - 1)(\sin x + 2) = 0$

The following lesson form and didactic modalities are anticipated for this phase of the course of the lesson.

A. Lesson form

a) *Didactic ground forms.*
Conversation and example.
b) *Methodological principle.*
Inductive and deductive.
c) *Principle of ordering content.*
Punctual and spiral.

B. Didactic modalities

a) *Principles of actualization*
(i) *Stating problem.*
Guided activity.
(ii) *Formulating problem*
Self activity.
b) *Modes of learning.*
Sensing and perceiving.
c) *Teaching aid.*
Blackboard.

Exposing the new

The difference as well as the similarity between solving a trigonometric and an algebraic equation can be shown to the pupils with a good example. From the statement of the problem, the following steps therefore are necessary in exposing the new.

New theory

(i) Defining the original equation as an equation with only one unknown.
a) Known identity.

(i) $2 \cos x = 3 \tan x.$

a) Put $\tan x = \frac{\sin x}{\cos x}$
in equation (i).

$\therefore 2 \cos x = \frac{3 \sin x}{\cos x}$

b) Simplifying: Least Common Multiple.

b) $2 \cos^2 x = 3 \sin x$

c) Known identity.	c) Put $\cos^2 x = 1 - \sin^2 x$ in (b): $\therefore 2(1-\sin^2 x) = 3\sin x$.
d) Simplifying. Remove brackets and bring all terms to one side.	d) $2 - 2\sin^2 x = 3\sin x$. $-2\sin^2 x = 3\sin x+2=0$
e) Multiply by (-1).	e) $2\sin^2 x+3\sin x-2=0$
(ii) Factor the *trinomial.*	(ii) $(2\sin x-1)(\sin x+2) = 0$.
Foreknowledge for determining angle sizes,	
a) Each term of the equation now can be equated with *zero.*	a) $2\sin x-1=0$, $2\sin x=1$, $\sin x=1/2$ | $\sin x+2=0$, $\sin x=-2$, unsolv.
b) Determine angle sizes from tables.	$x=30^0+k360^0$ or $x=150^0+k360^0$; $x=30^0$; 150^0; -330^0; -210^0.

From the above it is clear that the new insights are made meaningful on the basis of the foreknowledge and then are integrated with each other into a new or "different" structure.

The following lesson form and didactic modalities are anticipated for this phase.

A. Lesson form

a) *Didactic ground form(s).*
Conversation and example.

b) *Methodological principles.*
Inductive and deductive.

c) *Principle of ordering content.*
Punctual, concentric.

B. Didactic modalities

a) *Principles of actualization*
Guided activity.
Guided tempo.

b) *Modes of learning.*
Perceive and think.

c) *Teaching aid.*
Blackboard.

Actualizing the learning content

The following example can be done together with the pupils to implement the learning content: Solve for x if 5 cos x + 12 cosec x = Cot x if x € [-360⁰; 360⁰.]

This example is worked through with the pupils to provide security and confidence regarding *simplifying* the equation by applying known identities as well as because through simplification the equation is changed to a form where the factors can be acquired through *grouping*.

The following is given as a concrete illustration of this part-structure.

Example that is worked through with the pupils:

(i) Example	(i) $5 \cos x + 12 \operatorname{cosec} x = 15 + 4 \cot x$.
(ii) Substitution Identity: cosec x =	(ii) $5 \cos x + \frac{12}{\sin x} = 15 + \frac{4 \cos x}{\sin x}$ $5 \cos x \sin x + 12 = 15 \sin x + 4 \cos x$. $5 \cos x \sin x - 4 \cos x - 15 \sin x + 12 = 0$. $\cos x(5 \sin - 4) - (5 \sin x - 4) = 0$.
(iii) *Factors:* Grouping.	(iii) $(5 \sin x - 4)(\cos x - 3) = 0$.

$5 \sin x - 4 = 0$	$\cos x - 3=0$.
$5 \sin x = 4$.	
$\sin x = 4/5$	$\cos x = 3$.
$\sin x = .8$	unsolv.

(iv) Read the size of the angles from the tables.

(iv) $x = 53^0 8' + k360^0$
or $x = 126^0 52' + k360^0$
$x = 53^0 8'$; $126^0 52'$;
$-306^0 52'$; $-233^0 8'$.

The following lesson form and didactic modalities are anticipated here.

A. Lesson form

a) *Didactic ground forms*
Conversation and example.

b) *Methodological principle*
Deductive.

c) *Principles for ordering contgent*
Punctual, linear.

B. Didactic modalities

a) *Principles of actualization*
Guided and self activity.
Guided tempo.

b) *Modes of learning*
Perceive, think and
Practice (imitate).

c) *Teaching aid*
Blackboard.

Functionalizing new insights

The following examples can be worked through by the pupils themselves to practice the newly acquired insights and integrate them with their knowledge on hand.

(i) *First example that must be worked through by each pupil*
Solve for x if $6 \cos^2 x = 10 + 11 \sin x$ if $x \in [-360^0; 360^0.]$ With this example the aim is to *practice simplifying* by applying *identities* and then analyzing the resulting factors. Also, there is a striving for an integration of the new with the already available insights regarding matters such as *removing brackets, solving quadratic equations* and the terrain of compiling values of sin x (namely: $-1 \leq \sin x \leq 1$).

Concrete illustration of the example:

(i) Example.

(i) $6 \cos^2 x = 10 + 11 \sin x$.

(ii) Simplifying: Quadratic identity.

(ii) $6 (1 - \sin^2 x) = 10 + 11 \sin x$.

$6 - 6 \sin^2 x = 10 + 11 \sin x.$

(iii) Removing brackets.

(iii) $-6 \sin^2 x - 11 \sin x - 4 = 0.$

(iv) Write in the general form.

(iv) $6 \sin^2 x + 11 \sin x + 4 = 0.$

(v) Analyze into factors:

(v) $(2 \sin x + 1)(3 \sin x + 4) = 0.$

(vi) Determine possible angle values.

$2 \sin x = -1$ $\sin x = -1/2$ $x = 210^0 + k360^0$ or $x = 330^0 + k360^0$	$3 \sin x + 4 = 0.$ $3 \sin x = -4$ $\sin x = -4/3$
(vi) $x = 210^0; 330^0; -150^0; -30^0.$	unsolv.

(vii) The *terrain* of sin x.

(ii) *Second example that must be worked through by each pupil*
Solve for x if $3 \cos x - 2 = 3 - 2 \cot x$ and $\in[-360^0; 360^0.]$ With this example the aim is to *practice simplifying* by applying *known identities* and analyzing into factors by *grouping*. Also, it creates an opportunity for integrating the new with the already existing insight regarding solving equations in a fraction form. The following is an attempt to concretely illustrate this aim.

(i) Second example.

(i) $3 \cos x - 2 \operatorname{cosec} x = 3 - 2 \cot x.$

(ii) Simplifying: Known identities:

(ii)

$$3 \cos x - \frac{2}{\sin x} = 3 - \frac{2 \cos x}{\sin x}$$

(iii) Make use of the Lowest Common Multiple to simplify the fraction form.

(iii)

3 cos x sin x − 2 =
3 sin x − 2 cos x.
3 cos x sin x − 3 sin x +
2 cos x − 2 = 0.

(iv) Grouping

(iv)

3 sin x (cos x − 1) +
2(cos x − 1) = 0.
(cos x − 1)(3 sin x + 2) = 0.

cos x − 1 = 0 cos x = 1	3 sin x + 2 = 0 3 sin x = −2

(v) Read out the angle values.

(v)

x = 0° + k360° ∴ x = 0°; −360°; +360°	$\sin x = \frac{-2}{3}$ sin x = 0,6667 x = 221°49′ +k360° of x = 318°11′+ k360° x=221°49′; −138°11′; 318°11′; −41°49′.

The following lesson form and didactic modalities are anticipated here.

A. Lesson Form

a) *Didactic ground form*
Assignment.

b) *Methodological principle*

B. Didactic modalities

a) *Principle of actualization*
Self-individualization toward own tempo.

b) *Modes of learning*

Inductive. Think, remember.

c) *Principle of ordering content*
Punctual, concentric.

c) *Teaching aid*
Textbook.

Evaluating

A number of good problems can be presented to the pupils on a test to evaluate their level of achievement.

(i) *First problem*

Solve for x if tan x = 3 cot x and x €[-360^0; 360^0.] To guide the pupils to a degree, initially it is only expected that they fill in the blank spaces (missing information) in each step.

The explanation can be seen in the following examination item.

Complete: tan x = 3 cot x.

∴ _______ = 3 ________ (identity)

$\sin^2 x = 3\cos^2 x$.

$\sin^2 x = 3$ (______) (identity).

$4\sin^2 x = 3$.

$\sin^2 x = 3/4$.

sin x = _____ (factors).

Sin x = ______ sin x = ______

x = x =

This question is posed to evaluate if the child can simplify and determine the possible angle values. Also it is possible that the child already stagnates moderately at this early phase of the test in the absence of closer guidance.

(ii) *Second test problem*

Solve for x if $6\sin^2 x - 14\cos^2 x = 13\sin x - 16$ and x €[-360^0; 360^0.]

With this example the follow steps of thinking are anticipated:

a) The child can possibly try to group because the equation is made up of four terms.

b) The child who does not have insight into simplification can follow an incorrect way by expressing the $\sin^2 x$ in terms of $\cos^2 x$. This will obstruct the matter.

c) Insightful command of the quadratic identities and factors (trinomial) in order to simplify the trigonometric equation.

d) The correct determination of the angle values within the given interval and that satisfies the equation.

Only for the sake of a concrete illustration and explication of the theory, we provide the problem completely worked out.

(i) *Second test problem:*

(i) $6 \sin^2 x - 14 \cos^2 x = 13 \sin x - 16.$

(ii) Substitution of the quadratic identity. $\cos^2 x = 1 - \sin^2 x$.

(ii) $6 \sin^2 x - 14(1 - \sin^2 x) = 13 \sin x - 16.$

$6 \sin^2 x - 14 + 14 \sin^2 x = 13 \sin x - 16.$

(iii) Write in a general form.

(iii) $20 \sin^2 x - 13 \sin x + 2 = 0.$

(iv) Analyze into factors.

(iv) $(5 \sin x - 2)(4 \sin x - 1) = 0.$

$5 \sin x - 2 = 0$	$4 \sin x - 1 = 0$
$5 \sin x = 2$	$4 \sin x = 1$
$\sin x = \frac{2}{5}$	$\sin x = \frac{1}{4}$

(v) Determine the possible angle values.

(v) sin x = 0,4	sin x = 0,25
x = 23°35′ + k360°	x = 14°29′ + k360°
of x = 156°25′ +k360°	of x = 165°31′ +k360°
∴ x = 23°35′; 156°25′ −203°35′; −336°25′	x = 14°29′; 165°31′; −194°29′; −345°31′

With these two examples there is an attempt to evaluate the essences of the learning aim, namely, simplifying the equation, determining the size of the angles and the skillfulness with the methods.

For this final phase of the course of the lesson the following lesson form and didactic modalities possibly can be planned.

A. Lesson form

a) *Didactic ground form*
Assignment.

B. Didactic modalities

a) *Principle of actualization*
Self- individualizing.

b) *Modes of learning*
Think, remember.

c) *Teaching aid*
Duplicated exam.