

(iii) Make use of the Lowest Common Multiple to simplify the fraction form.

(iii)

$$\begin{aligned} 3 \cos x \sin x - 2 &= \\ 3 \sin x - 2 \cos x. \\ 3 \cos x \sin x - 3 \sin x + \\ 2 \cos x - 2 &= 0. \end{aligned}$$

(iv) Grouping

(iv)

$$\begin{aligned} & 3 \sin x (\cos x - 1) + \\ & 2(\cos x - 1) = 0. \\ (\cos x - 1)(3 \sin x + 2) &= 0. \\ \left| \begin{array}{l} \cos x - 1 = 0 \\ \cos x = 1 \end{array} \right| & \left| \begin{array}{l} 3 \sin x + 2 \\ = 0 \\ 3 \sin x \\ = -2 \end{array} \right. \end{aligned}$$

(v) Read out the angle values.

(v)

$$\begin{array}{l} x = 0^\circ + \\ k360^\circ \\ \therefore x = 0^\circ; \\ -360^\circ; \\ +360^\circ \end{array} \left| \begin{array}{l} \sin x = \frac{-2}{3} \\ \sin x = \\ 0,6667 \\ x = 221^\circ 49' \\ +k360^\circ \\ \text{or } x = \\ 318^\circ 11' + \\ k360^\circ \\ x = 221^\circ 49'; \\ -138^\circ 11'; \\ 318^\circ 11'; \\ -41^\circ 49'. \end{array} \right.$$

The following lesson form and didactic modalities are anticipated here.

### A. Lesson Form

### B. Didactic modalities

a) *Didactic ground form*  
Assignment.

a) *Principle of actualization*  
Self-individualization  
toward own tempo.

b) *Methodological principle*

b) *Modes of learning*

Inductive.

Think, remember.

c) *Principle of ordering content*  
Punctual, concentric.

c) *Teaching aid*  
Textbook.

### ***Evaluating***

A number of good problems can be presented to the pupils on a test to evaluate their level of achievement.

#### (i) *First problem*

Solve for  $x$  if  $\tan x = 3 \cot x$  and  $x \in [-360^\circ; 360^\circ]$ . To guide the pupils to a degree, initially it is only expected that they fill in the blank spaces (missing information) in each step.

The explanation can be seen in the following examination item.

Complete:  $\tan x = 3 \cot x$ .

$$\therefore \underline{\hspace{2cm}} = 3 \underline{\hspace{2cm}} \text{ (identity)}$$

$$\sin^2 x = 3 \cos^2 x.$$

$$\sin^2 x = 3 \underline{\hspace{2cm}} \text{ (identity).}$$

$$4 \sin^2 x = 3.$$

$$\sin^2 x = 3/4.$$

$$\sin x = \underline{\hspace{2cm}} \text{ (factors).}$$

$$\sin x = \underline{\hspace{2cm}} \quad \sin x = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}} \quad x = \underline{\hspace{2cm}}$$

This question is posed to evaluate if the child can simplify and determine the possible angle values. Also it is possible that the child already stagnates moderately at this early phase of the test in the absence of closer guidance.

#### (ii) *Second test problem*

Solve for  $x$  if  $6 \sin^2 x - 14 \cos^2 x = 13 \sin x - 16$  and  $x \in [-360^\circ; 360^\circ]$

With this example the follow steps of thinking are anticipated:

a) The child can possibly try to group because the equation is made up of four terms.

b) The child who does not have insight into simplification can follow an incorrect way by expressing the  $\sin^2 x$  in terms of  $\cos^2 x$ . This will obstruct the matter.

c) Insightful command of the quadratic identities and factors (trinomial) in order to simplify the trigonometric equation.

d) The correct determination of the angle values within the given interval and that satisfies the equation.

Only for the sake of a concrete illustration and explication of the theory, we provide the problem completely worked out.

(i) *Second test problem:*

$$(i) \quad 6 \sin^2 x - 14 \cos^2 x = 13 \sin x - 16.$$

(ii) Substitution of the quadratic identity.

$$\cos^2 x = 1 - \sin^2 x.$$

$$(ii) \quad \begin{aligned} 6 \sin^2 x - 14(1 - \sin^2 x) &= 13 \sin x - 16. \\ 6 \sin^2 x - 14 + 14 \sin^2 x &= 13 \sin x - 16. \end{aligned}$$

(iii) Write in a general form.

$$(iii) \quad 20 \sin^2 x - 13 \sin x + 2 = 0.$$

(iv) Analyze into factors.

$$(iv) \quad (5 \sin x - 2)(4 \sin x - 1) = 0.$$

$$\begin{array}{l|l} 5 \sin x - 2 = 0 & 4 \sin x - 1 = 0 \\ 5 \sin x = 2 & 4 \sin x = 1 \\ \sin x = \frac{2}{5} & \sin x = \frac{1}{4} \end{array}$$

(v) Determine the possible angle values.

(v) $\sin x = 0,4$	$\sin x =$
	$0,25$
$x = 23^{\circ}35' +$	$x = 14^{\circ}29'$
$k360^{\circ}$	$+ k360^{\circ}$
of $x = 156^{\circ}25'$	of $x =$
$+k360^{\circ}$	$165^{\circ}31'$
	$+k360^{\circ}$
$\therefore x = 23^{\circ}35';$	$x = 14^{\circ}29';$
$156^{\circ}25'$	$165^{\circ}31';$
$-203^{\circ}35';$	$-194^{\circ}29';$
$-336^{\circ}25'$	$-345^{\circ}31'$

With these two examples there is an attempt to evaluate the essences of the learning aim, namely, simplifying the equation, determining the size of the angles and the skillfulness with the methods.

For this final phase of the course of the lesson the following lesson form and didactic modalities possibly can be planned.

#### **A. Lesson form**

- a) *Didactic ground form*  
Assignment.

#### **B. Didactic modalities**

- a) *Principle of actualization*  
Self- individualizing.
- b) *Modes of learning*  
Think, remember.
- c) *Teaching aid*  
Duplicated exam.