

- a) (i) *Meaningful unlocking of everyday language usage:*  
side, opposite side, upper side, opposite, opposite to, opposite each other, etc.  
(This involves spatial orientation and the location of one matter with respect to another.)

(ii) The concept “opposite” in geometric contexts.

(iii) A formal description of the concept “opposite”:

(1) When two lines intersect each other, two pairs of opposite angles are formed at the point of intersection.

(2) The size of each pair of opposite angles can differ.

(3) A pair of opposite angles always is equal.

### Theory

b) Proof of the proposition:

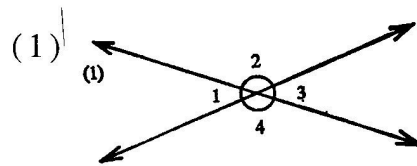
(i) Diagrammatic representation

1. Shops that are opposite each other on both sides of the same street.

2. Opponents in the two opposite corners of a wrestling or boxing ring.

3. The opposite angles of rectangles and parallelograms are equal.

(ii) *Example:* Pairs that cross such as lines that intersect.



$$\begin{array}{cc} \hat{O}_1 & \text{and} & \hat{O}_3 \\ \hat{O}_2 & \text{and} & \hat{O}_4 \end{array}$$

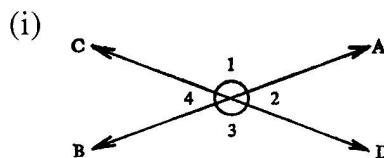
(2)  $\hat{O}_1$  and  $\hat{O}_3$  are acute angles.

$\hat{O}_2$  and  $\hat{O}_4$  are obtuse angles.

$$\begin{array}{cc} \hat{O}_1 = \hat{O}_3 \\ \hat{O}_2 = \hat{O}_4 \end{array}$$

### Example

*Illustration*



*Proof*

(ii) Linking up with fore-knowledge re *straight angles*.

$$\begin{aligned} \hat{O}_1 + \hat{O}_2 &= 180^\circ \\ \hat{O}_2 + \hat{O}_3 &= 180^\circ \end{aligned}$$

(iii) The “equality” concept.

$$\hat{O}_1 + \hat{O}_2 = \hat{O}_2 + \hat{O}_3$$

(iv) The concept “common angle”.

(iv) But  $\hat{O}_2$  is included on both sides.

$$\begin{aligned} \hat{O}_1 + (\hat{O}_2 - \hat{O}_2) &= \\ \hat{O}_3 + (\hat{O}_2 - \hat{O}_2) &= \\ \hat{O}_1 &= \hat{O}_3 \end{aligned}$$

(v) *Control (check, verify)*

(v) Only thus can it be proven that  $\hat{O}_1 = \hat{O}_3$ .

## II. Lesson form

### a) *Ground form*

(i) *The concept “opposite angles”*

In terms of a *conversation* about the examples mentioned (shops, ring, pairs and lines) there is a meaningful linking up with the everyday language usage and a push through to the formal definition of the concept opposite angles in a geometric context.

(ii) *The proof*

For the exposition of the formal proof of the proposition the same ground forms are chosen again, namely conversation and example.

### b) *Methodological principles*

(i) *Elucidating the concept: Inductive*

To be able to arrive at the formal definition of the concept opposite we begin with everyday examples and work through to the general definition of proposition.

(ii) *The proof: deductive*

In proving the proposition we are going to work deductively where from the formal description of the proposition there is a push through to particular examples (problems).

c) *Principle of ordering: Punctual*

Where the proposition is taken as the point of departure for the presentation this always is an indication that a punctual ordering can be successful.

Here one goes from the difficult and complex proposition in search of a suitable example that can illustrate further its various characteristics. Each time there is a return to the formulation of the proposition as the point of departure. We provide a brief example:

- (i) *General postulate:* If two lines intersect each other the opposite angles are equal.
- (ii) *First characteristic:* The concept opposite
- (iii) *Second characteristic:* Two lines intersect each other.
- (iv) *Third characteristic:* Equal angles.
- (v) *Fourth characteristic:* Relationship among the first three characteristics.

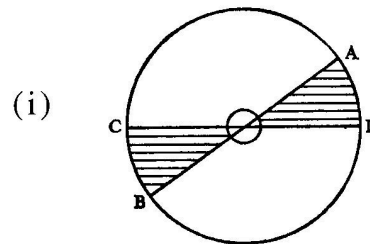
### III. Didactic Modalities

a) *Principle of actualization*

*Guided activity*

b) *Modes of learning*

(i) *Sensing.* By making use of a cardboard model where the size of the angle can be changed and a particular color effect shown, a pupil can acquire a first, global idea of the matter.



Line AB moves around O.

(ii) *Perceiving.* The totality view now must be broken open and the problematic isolated.

