CHAPTER 1
GENERAL AND PARTICULAR PROBLEMS IN TEACHING MATHEMATICS

1.1 INTRODUCTION

During the past few years, in most countries there has been an attempt to change and renovate teaching. In accordance with the demands of the time, real steps have been taken to modernize the teaching of mathematics and not only with respect to the contents but also regarding the aims and forms of presentation. Because research in this subject area today has become so comprehensive, there is more knowledge at our disposal at a faster rate than its value can be estimated. Consequently, sometimes provision is made for teaching these changes in unaccountable ways.

Along with the great knowledge "explosion" of the past decades in almost every subject area, impressive progress has also been made in the area of mathematics teaching. This progress has allowed didacticians in this country to re-assess the contents as well as the form of their teaching. This has resulted in a clear shift in emphasis from memorized knowledge and mechanistic manipulations. Forms of teaching and learning that initiate true insight and refined concept formation, clear perceiving and greater proficiency now are planned.

Proficiency in the meaningless manipulation of numbers and symbols must make an important place for insight into the meaning and structure of basic rules, axioms and formulas. Now the aim is to deal insightfully with these number and symbol relations in new situations. This renovation is directed to drastically diminishing the earlier unfruitful memorization and imprinting of learning materials contents, relations and methods.

However, every mathematics teacher remains explicitly aware that a minimum amount of ready knowledge and a particular level of proficiency in mathematical manipulations are indispensable and are the basis for each mathematical operation or problem solution. Pupils who have memorized mathematical theory without insight will often be able to reproduce it. However, as soon as they are given the task of applying such a theory, e.g., to solving new problems,

fruitful transfer and real achievement will occur only if the matter was acquired with insight. Real insight and refined understanding can be used only if the pupils have the opportunity to themselves discover the essentials, relationships and rules.

The cultural demands of the past few decades on purposefully educating (upbringing) and teaching have increased tremendously regarding their breadth and depth with the consequence that increasingly more is asked of the child without being able to provide extra time. With such a deluge of learning material, a teacher in his planning of learning contents has to differentiate between what appears to be **essential** or **incidental, invariant** or **changeable.**

On the basis of the problem as stated, it appears that the reduction and disclosure of the essentials of a particular slice of reality, and the design of an accountable lesson structure, within which the actualization of such unlocking and self-discovery is possible, can possibly make a contribution to the pupil attaining real insight and a formation of clear concepts.

The concept "reduction of learning material", in general, and "reduction of mathematics learning material", in particular, as well as their place and meaning in designing an accountable lesson structure are the focus of this study. Before truly realizing the problems of teaching mathematics when real insight and clear concept formation are striven for, it is necessary to more clearly delimit and formulate these problems. By distinguishing between general and particular problems, there is an ordering by which particular ways of teaching and learning become relevant to making the above-mentioned aims possible.

1.2 GENERAL PROBLEMS

In the past, in secondary schools great emphasis was placed on commanding a large amount of factual knowledge and on the mechanistic application of particular methods. Evaluation in such classrooms, as well as on common examinations, was focused too much on testing for readiness. Along with the renovation and expansion that occurred in the past few years in the separate fields of science and in the various specializations of mathematics, a deluge of learning material became an actual problem and, on the other hand, a striving for completeness no longer was possible. Any striving for completeness occurred at the cost of originality. The

consequence was that the pupils of mathematics were forced, on the basis of their superficial knowledge, to adopt "thinking patterns" and for solving problems to firmly grasp practiced recipe methods and algorithms.

Hence, today there is a search for ways and forms of teaching and learning that allow for insight and clear concept forming, greater mobility and suppleness in the disclosure and implementation of methods of solution to attain their rightful place in the future. The emphasis shifts from memorizing a great deal of ready quantitative knowledge and the imitation of algorithms to greater insight into the essentials of the concepts and structures of such an area of reality. That is, the teacher has to remain directed to the fundamental concepts, relationships and general methods in each particular subject area that leads to the greatest learning effect. By a particular focus on the most purposeful use of the different contents of the matter, the teacher can succeed in designing problem situations in which the child can participate by perceiving, thinking and remembering

Regarding this, J. J. Dreyer[1] makes the following important point: "In the past pupils have been taught **how** to perform a mathematical operation. For this purpose they were given a 'rule', and emphasis was directed towards the achievement of results by strict observance of the 'rule', whether or not they understood what reason was behind it. The Modern Approach stresses the fundamental importance of an **understanding of concepts**. **Why** is now more important than **How"**.

By performing a reduction, the teacher can disclose the essentials of the concept or subject to himself and then incorporate them into his lesson aim. In planning a lesson structure it now is necessary to shape the learning aim into a problem for the child. By further designing the lesson situation in such a way that there is a meaningful linking up with the child's foreknowledge the teacher can insure to, a greater degree, that the pupils themselves **can** and **will** search for a solution. Therefore, here the emphasis is more on insightful learning and less on learning devices and achieving apparent results.

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[1] Dreyer, J.J.,*Modern mathematics for South African Schools. Standard 7 (Grade 9).* Preface.

A real perspective on the aims that today are held for mathematics teaching can only be acquired after the above-mentioned course of learning is looked at. The compilers of a common basic syllabus for mathematics require that the following general aims be pursued. (These general aims are quoted as they appear in the appendix to Spectrum 42, July 1966).

(i) To acquaint the pupils with the mathematical world within which everyday persons have to measure and count;
(ii) to make pupils proficient in the use of mathematical knowledge and methods for solving questions (problems) in order to be able to find their way in various of branches of vocational life;
(iii) to cultivate in pupils correct, accurate and systematic work methods;
(iv) to give the pupils a basic insight, grasp and knowledge of those basic mathematical principles that prepare them for further study in mathematics and in the natural sciences;
(v) to cultivate in the pupils a love for and interest in the further study of mathematics and the natural sciences.

From the above it is clear that the emphasis has shifted from a mechanistic manipulation and thoughtless imitation to the acquisition of insight and to clear concept formation. Insight into the structure and unique nature of mathematics, as well as the mutually related and corresponding knowledge of its part-specialties such as fruitful methods of solution are an important part of this study. When insightful interpretations, formation of clear concepts, integration and the discovery of fruitful methods of solution are the aims, only a method of teaching that involves the pupils in their totality has any hope of succeeding. Thus, a teacher has the task of planning and designing his lesson situation such that there will be opportunities for original experiences, experiences of relevance to reality and independent discoveries of insightful learning, unambiguous interpretations, clear concept forming and a guiding to logical deductions.

To try to accomplish the above aims the teacher, in his preparation and planning of a specific lesson situation, has to be able to distinguish clearly and unambiguously between what, as an essential of the contents, is omitted from his exposition of the new and that must be presented anew, and what is not essential and must or can be left out of account. Such generally valid pronouncements and choices only can be reached if the teacher himself has reduced the

learning contents and searched for what appears to be fundamental, necessary and invariant. The creation of a fruitful moment for learning only can be planned scientifically if, beforehand, the teacher has first clearly delimited the essentials of the topic for himself. This procedure, where the essence of the matter is searched for, is known in didactic theory as the "reduction of the learning material".

Now, to arrive at a systematic and ordered statement of the problem based on the above views, there is an attempt to disclose a number of specific problems and to arrange them in accordance with our aims for each phase of the course of a lesson.

1.3 SPECIFIC PROBLEMS

1.3.1 Problems with the beginning of the lesson

In order to involve the pupils in the learning event, the first requirement is that the design of the situation be meaningful to them. That is, a link has to be sought between the new contents and the pupils' experiential worlds, and the learning material has to be presented in such a way that they are enticed to participate. The problems of the beginning of the lesson are summarized in the following two categories: Those problems related to linking up with foreknowledge and questions regarding stating the problem.

(a) Problems with the lesson beginning

To try to link up meaningfully with the children's experiential worlds it is necessary to take into account the level of becoming and learning readiness of the class in order to be able to decide what foreknowledge has to be actualized. Here one distinguishes between problems with the child and problems with the learning material.

Problems with the child: When the child's potentialities and readiness are not taken into account the beginning of the lesson can lead to general problems, i.e., errors the entire class is going to make, but also particular problems because each child differs from each other. Thus, the teacher's introductory orientation can be given at too high a level or things can be assumed to be obvious but which the pupils are not acquainted with. Where the beginning is on too easy a level or is unnecessarily concrete or confused, the interaction and harmony between the pupils'

potentialities and readiness is disturbed and this can give rise to boredom and frustration.

Problems with the learning material: (i) If, in his preparation, the teacher has not reduced the learning material to its essentials he can confuse the child with his presentation because:

* He has not worked purposefully and, therefore, cannot help the pupils distinguish between details and what is essential to the matter;

* He, therefore, also is not in a position to select examples that already have sense and meaning for the child.

(ii) Since the foreknowledge and basic structure that are necessary for insight into the new concepts are not clearly recalled and made explicit, in first exposing the new contents, and later in its application, the teacher can discover that the pupils have deficiencies and uncertainties about the essentials of the matter. These resistances and deficiencies in the child's basic experiential world, that the child undergoes so early in the course of learning, can lead to an unnecessary waste of time, interruption, confusion and stagnation.

(b) Problems with stating the problem

Lived-experiencing the problem: The child's lived-experience of the problem differentiates itself in two ways: First on a more pathic-affective level as in the strangeness, unfamiliarity, and foreignness of the matter, which during sensing, as a mode of learning, manifests itself in a global understanding of the matter. About this, Sonnekus[2] says: "Sensing is a **totality act** of the person, or differently stated, a totality-in-communication with the world". On the other hand, as in mainly cognitive perceptions and disclosures, there is a search for the core and objective essentials of the object. In this reducing activity of perceiving, the problem is more clearly defined and it becomes a problem-for-me (the learning person).

Giving meaning: Along with sensing and perceiving there also is an attempt to give meaning to what is viewed by ordering and schematizing it. The matter first becomes a truly meaningful problem when the learner is able to relate it to his

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[2] Sonnekus, M.C.H., *Die leerwereld van die kind as beleweniswereld*, p. 63c.

foreknowledge. After that he realizes, however, that what he knows about the matter has gaps and is an incomplete structure to be filled. Along with giving meaning, the first impetus for the child is to enter into the matter with the aim of trying to find a solution to the problem through attending to it.

Motivating: The teacher also has the task of so presenting the learning aim that its essentials show a formative quality and strongly appeal to and motivate the child such that there is a clearly observable indication of greater attention, rapt focusing and real interest.

The "New Seminar for Pedagogics"[3] at the University of Amsterdam states the following preconditions for stating the problem:

* "The problem has to be in the pupils' sphere of interest".
* "All activity should lead the pupils to a solution".
* "The pupils have to master the solutions they have found".

Own problem: The new contents should be presented in such a way that the problem is evident to the child on the basis of the teacher representing then so they can be interpreted more easily and formulated as one's own problem.

Searching for a solution himself: The teacher has to present the problem such that it demands that the pupils themselves will seek a solution, otherwise they are forced, on the basis of defective insight, to participate mechanistically in the problem solving event. This only can lead to apparent results.

1.3.2 Problems (methods) with offering the new contents

No methods can guarantee a fruitful lesson situation. Each teacher must try in his presentation to anticipate the most effective methods that will lead to insightful learning. The following problems of offering the new contents are distinguished: the problem of mechanical imitation and the problem of premature generalization.

(a) The problem of mechanical imitation

[3] New Seminar for Pedagogics at the University of Amsterdam, Communication No. 74, p. 24b.

In practice we find that young and inexperienced teachers often blindly apply the methods that they themselves have experienced in school. This so-called "recipe-like" form of teaching is not always the most recent and fruitful of didactic views. We think here of the consistent use of the deductive method in teaching geometry, which for many years has been accepted by teachers without reservation for each lesson.

(b) The problem of premature generalization

Because more experienced teachers accept certain things as obvious, it can happen that there are premature generalizations and a leading of pupils to rules and formulas long before they have arrived at insights into the essential characteristics or structure of the concept by experiencing varied situations and adequate examples, i.e., in an inductive way. In this way the learning person is forced to learn mechanically (by rote). Thus, there remains for him (the child) just one way out, namely, to memorize the new concepts, rules, theorems and formulas without acquiring insight into building up the structure and into the meaning of the matter.

Van Rooy[4] directs the following warning against this: "Too often it is forgotten that the contemporary form of our science is the end product of years of cooperative refinement by the most gifted thinkers"

1.3.3 Problems with controlling (checking) the new

The success obtained with programmed instruction most certainly, to a large degree is that, as far as possible, the program is attuned to checking each act of thinking and unique achievement of each pupil. Although certain objections can be raised against the behaviorist foundation of these particular aims, namely, **immediate reinforcement** of the pupil, still it leads indirectly to the fact that each misunderstanding and error, incorrect interpretation and blockage are immediately evident to the pupil. In this self-discovery of his potentialities as well as his deficiencies lies the formative value of the instruction.

The child's checking is related to the act of guidance. At the beginning of each year and with the presentation of each "new"

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[4] Van Rooy, D.J., *Die beginsels van die onderwys van Algebra en Rekenkunde*, p. 4b.

topic, learning must be checked as often as possible. As the child acquires more insight, the rhythm of the checking can be decreased accordingly.

The experienced teacher is able to anticipate which parts of the subject contain difficult relationships, abstract concepts and refined methods of solution that are not easily understood by the pupils. Therefore, he selects good examples to illustrate the expected problems and possible stumbling blocks. The **inexperienced** teacher can only find out about these **essential** aspects through good preparation and in this way purposefully pause and give special attention to them.

1.3.4 Problems of application

In this phase of the lesson usually the problem arises of what examples to use and how many. This problem differentiates itself further and is treated in more detail below.

(a) The choice of unimaginative and unrealistic examples that have no clear use or that appear so easy that they do not direct an appeal to the child.
(b) Excessive drill work and repetition only for the sake of the repetition itself give rise to frustration.
(c) Foreign to life and abstract assignments contribute to the child constructing two worlds, namely that of the school and that outside of the school.
(d) It often happens that fundamental concepts and operations are learned in isolation without becoming integrated with the foreknowledge as a larger functional unity.

1.3.5 Problems of evaluation

According to Van Hiele[5] there only can be an insightful activity when the pupil "intentionally acts adequately in new situations". Requirements that meet this precondition are summarized in three groups.

(a) By asking about definitions, formulas, theorems and rules one can test whether the pupils know their theory. However, this is not yet a test of insight and understanding.

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[5] Van Heile, P.M., *De problematiek van het inzicht*, p. 11b.

(b) By asking questions about where the newly acquired theory has to be applied to be able to reach a solution, the teacher can determine the extent to which the pupils have insight into the essentials of the matter.
(c) Assignments also can be given in which the child has to construct a new structure or relationship from his previous mastery.

A good test always has to deal with all three aspects. The teacher is only able to delimit and point out the **core learning material** so that it is insightfully mastered by all pupils and can be made their own by good preparation and thorough reduction of the contents. Only when pupils achieve on tests with respect to these contents can there be a move to supplemental and orienting programs.

1.4 SUMMARY

All of the general problems regarding guidance to insightful interpretations, clearer concept formation and the discovery of good methods of solution, as well as the particular problems at the beginning, the course and the end of a lesson require careful reflection and scientific research before an accountable lesson structure can be constructed. Therefore, to be able to view the relationship of these problems within the totality of the lesson structure in the right perspective and to be able to make meaningful comments, it is necessary to look more closely at certain didactic findings.

Solutions to the mentioned problems should be sought in didactic theory so that there is less need in future planning and designing of lesson situations to rely on chance. Thus, there should be a search in didactic theory for findings regarding "the reduction of learning material" and its place and value in the lesson structure. Certainly, therefore, it is appropriate to proceed in the second chapter to a discussion of the question of "the reduction of learning material" and the possibilities and meanings this opens for practice to the extent that it can contribute to helping solve the above **problems**.

However, before the essentials of the concept "reduction of learning material" can be penetrated, it is first important to say something about **learning material** and the selection of suitable **learning contents.**

There also is an attempt to answer the question of what reduction is. To be able to venture an answer there necessarily is a search for connections with the phenomenological method that essentially is a method of reduction. Therefore, the phenomenological method and opinions of Husserl and others are considered with the aim of explaining and introducing particular concepts.

Then, the concept reduction is put in its didactic context and elaborated as a didactic category that is its relevance for practice. Certainly one cannot neglect indicating that, in reality, a secondary situation such as the school actualizes this course in reverse. Where in general didactic theory, it is accepted that the form of teaching is universal and the contents particular, it is important to notice that in the planning of a lesson situation, the contents, as indicated in the different curricula, are firmly in reality. Proceeding from the learning contents and their essentials as learning aims, there then is a search for a lesson form that will bring out the best quality of the contents.

In the third chapter there is an attempt to indicate the place and value of reducing the learning material in the different phases of the lesson structure.

Also, to be able to give sense and meaning to all of these theoretical views and findings and allow them to have the most direct relevance for practice, we proceed to lesson preparation in mathematics in the last chapter.

In terms of topics from the mathematics syllabus there is an attempt to show how the theory of the previous chapters can be applied to practice. Also the examples can serve as **guidelines** for mathematics teachers who in the future are going to plan lesson situations themselves.