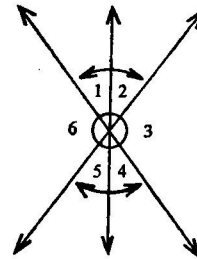


Because  $\hat{O}_1 + \hat{O}_2 = 180^\circ$   
 and  $\hat{O}_2 + \hat{O}_3 = 180^\circ$   
 $\therefore \hat{O}_2$  is common.

(iii) *Thinking*. Allow the child to point out opposite angles in a more complex figure.

(iii)



For example:  $\hat{O}_1 + \hat{O}_2 = \hat{O}_4 + \hat{O}_5$ .

(iv) *Remembering*. 1. A good example is worked through with the pupils to control their insights into the new.  
 2. A number of examples are worked through by the pupils themselves.

(iv) *Guided functionalizing*

Thus the lesson aim includes much more than the learning aim in the sense that the teacher for his presentation must anticipate and actualize certain supporting and supplementary principles, forms and modes of learning.

#### 4.2.3 Actualizing foreknowledge and reducing the learning material

a) *Theme*: (Algebra, Grades 11 and 12).  
 Proof of the remainder theorem.

b) *Reduction of the theme*:

### Illustration

The essence this theme is that  
When a polynomial in  $x$  is divided  
by  $(x - p)$  then the remainder is the  
same polynomial in  $p$ .

Suppose the polynomial  $t(x)$   
is divided by  $(x - p)$ . When  
we now accept that the  
quotient is going to be  $k(x)$   
and the remainder  $R$  (where  
 $R$  is zero) this means the  
following:

$$t(x) \cong (x - p) \times k(x) + R$$

Let  $x = p$ .

$$\begin{aligned} \therefore t(p) &= (p - p) \times k(x) + R. \\ &= 0 \times k(x) + R \\ &= 0 + R. \end{aligned}$$

*Disclosure:* The remainder immediately can be calculated by  
entering the polynomial in  $p$ .

#### c) Actualizing foreknowledge

The following foreknowledge as a supportive basis must be  
actualized beforehand.

### Theory

(i) *The notation system.*

(ii) The division as carried out  
with a polynomial as dividend and  
another polynomial of a lower  
value as divisor. Also the quotient  
must not be negative and there must  
be a remainder.

### Illustration

$$\begin{aligned} \text{(i)} \quad t(x) &= x^2 - 4 \\ t(2) &= (2)^2 - 4 \\ &= 4 - 4 \\ &= 0. \end{aligned}$$

$$\begin{array}{r} \text{(ii)} \quad \begin{array}{r} x + 2 \\ x - 5 \overline{) x^2 - 3x + 5} \\ \underline{x^2 - 5x} \phantom{+ 5} \\ 2x + 5 \\ \underline{2x - 10} \\ + 15 \end{array} \end{array}$$

$$\therefore x^2 - 3x + 5 \div (x - 5)(x + 2) + 15$$

Dividend = divisor x quotient +  
Remainder.

#### 4.2.4 Stating and formulating the problem and reducing the learning material

a) *Theme* (Trigonometry, Grades 11 and 12).  
Compound angles. Proof of the formulas  $\cos (A \pm B)$  and  $\sin (A \pm B)$ .

b) *Reduction of the theme*

##### Theory

(i) The essence of this theme can be carried back to the proof of the formula  $\cos (A + B) = \cos A \cos B - \sin A \sin B$ .  
The formulas for  $\cos (A + B)$  and  $\sin (A + B)$  all are derived from the above through substitution.

(ii) The essence of the proof of  $\cos (A - B) = \cos A \cos B + \sin A \sin B$  is in implementing the interval formula  
$$(AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2})$$
  
between two points  $A(x_1; y_1)$  and  $B(x_2; y_2)$  as defined in analytic geometry.

##### Illustration

$$\cos (A - B) = \cos A \cos B + \sin A \sin B.$$

(i) Substitute B with  $-B$ .

$$\begin{aligned} \therefore \cos [A - (-B)] &= \cos A \cos (-B) + \sin A \sin (-B) \\ \cos (A + B) &= \cos A \cos B - \sin A \sin B. \end{aligned}$$

(ii) Substitute A by  $(90^\circ - A)$ .

$$\begin{aligned} \cos(90^\circ - A - B) &= \cos(90^\circ - A) \cos B + \sin(90^\circ - A) \sin B. \end{aligned}$$

$$\begin{aligned} \cos [90^\circ - \overline{A + B}] &= \sin A \cos B + \cos A \sin B \\ \sin (A + B) &= \sin A \cos B + \cos A \sin B. \end{aligned}$$

(iii) Substitute B by  $-B$  in the formula  $\sin (A + B)$