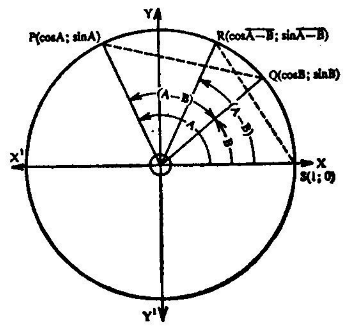
sin (A – B) = sin A cos (-B) + cos A sin (-B).

sin (A – B) = sin A cos B – cos A sin B.



PQ = RS (center angles Equal)

$PQ^2 = RS^2$

$\therefore (\cos B - \cos A)^2 + (\sin B - \sin A)^2 = (\cos \overline{A-B} - 1)^2 + (\sin \overline{A-B} - 0)^2$

$\therefore \cos^2 B - 2 \cos A \cos B + \cos^2 A + \sin^2 B - 2 \sin A \sin B + \sin^2 A = \cos^2 \overline{A-B} - 2 \cos \overline{A-B} + 1 + \sin^2 \overline{A-B}$

$\therefore (\sin^2 B + \cos^2 B) + (\sin^2 A + \cos^2 A) - 2 \cos A \cos B - 2 \sin A \sin B = (\cos^2 \overline{A-B} + \sin^2 \overline{A-B}) - 2 \cos \overline{A-B} + 1$

$\therefore 1 + 1 - 2 \cos A \cos B - 2 \sin A \sin B = 1 - 2 \cos \overline{A-B} + 1$

$\therefore 2 - 2(\cos A \cos B + \sin A \sin B) = 2 - 2 \cos \overline{A-B}$

$\therefore 2 \cos(A - B) = 2(\cos A \cos B + \sin A \sin B)$

$\therefore$ ***cos(A–B) = cosAcosB+ sinAsinB.***

c) *Stating and formulating the problem*

Illustration

(i) In terms of the foreknowledge actualized the pupil can realize that cos (A – B) is *not* equal to cosA – cosB but is equal to cosA cosB + sinA sinB. The question that arises is: How can this special case be shown to be generally valid?

(i) If $\hat{A} = 60^0$ and $B = 30^0$, $\hat{A} - B = 30^0$.

$\therefore \cos(A - B) = \cos 30^0$

$= \frac{\sqrt{3}}{2}$

$\cos A - \cos B = \cos 60° - \cos 30°$

$= \frac{1}{2} - \frac{3}{2}$

$= \frac{1 + \sqrt{3}}{2}$

$\cos A \cos B + \sin A \sin B$

$= \cos 60° \cos 30° + \sin 60° \sin 30°$

$= \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$= \frac{2\sqrt{3}}{4}$

$= \frac{\sqrt{3}}{2}$

The problem culminates in the pupils knowing:

(1) Where the solution cosA cosB + sinA sinB comes from. In other words how can it be demonstrated that cos (A – B) always is equal to cos A cosB + sinA sinB.

(2) The pupils are going to search alone for possible solutions to the other cases.

4.2.5 Exposing the new and reducing the learning material

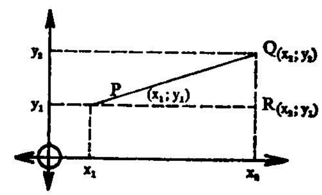
a)*Theme:* (Analytic Geometry, Grades 11 and 12).
The length of a line connecting two particular points.

b) *Reduction of the theme*
The essence of this theme is disclosed in the agreement shown with the Pythagorean theorem.

Theory

The essence is that the distance between any two points on a Cartesian plane can be determined by computing the hypotenuse when the right angle sides are known.

Illustration

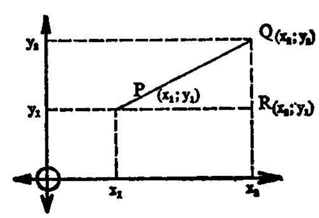


$PR = x_2 - x_1$
$RQ = y_2 - y_1$
$PQ^2 = PR^2 + RQ^2$ (Pythagoras)
$PQ^2 = (x_2-x_1)^2 + (y_2-y_1)^2$
$\therefore PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

c) *Exposing the new*
The exposition of the new exists in guiding the pupils to the new structure (length of PQ) by applying the Pythagorean theorem.

Exposition

Illustration



(i) On the basis of foreknowledge regarding the graphic:

(i) $PR = x_2 - x_1$
$QR = y_2 - y_1$
$PQ = ?$

(ii) The application of the Pythagorean theorem.

(ii) $PQ^2 = PR^2 + QR^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$