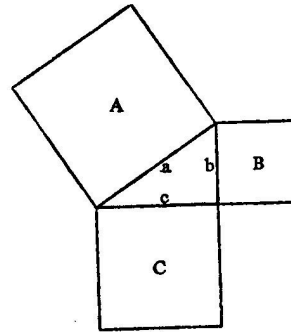


### Theory

The sum of the area the squares drawn on the shorter sides of a right triangle are equal to the area of the square drawn on the longest side.

### Illustration



$$A = B + C;$$
$$a^2 = b^2 + c^2$$

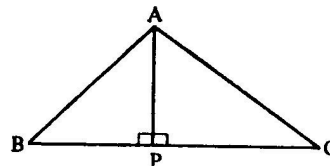
### c) Functionalizing the new insights

For a practicing of insights the following example can be carried out by the pupils themselves.

In triangle ABC,  $\hat{A} = 90^\circ$  and  $AP > BC$ . Prove that  $AB^2 - AC^2 = BP^2 - PC^2$ . With this example the application of the Pythagorean theorem is carried out.

### Theory

### Illustration



### Proof:

(i) Following the Pythagorean theorem.

$$(i) AB^2 = AP^2 + BP^2$$

(ii) Following the Pythagorean theorem.

$$(ii) \frac{AC^2 = AP^2 + PC^2}{AB^2 - AC^2 = BP^2 - PC^2}$$

The newly acquired geometrical insights can be integrated with already existing insights in Graphics, Trigonometry, Analytic Geometry and Arithmetic in terms of the following examples:

- |                          |   |
|--------------------------|---|
| (i) Graphics.            | (i) Draw the graph of<br>$x^2 + y^2 = 15$ .   |
| (ii) Trigonometry.       | (ii) In triangle PQR, $Q = 90^\circ$ .<br>Prove that $\sin^2 R + \cos^2 R = 1$ .  |
| (iii) Analytic Geometry. | (iii) A is the point $(x_1; y_1)$<br>and B is the point $(x_2; y_2)$ .<br>Derive a formula to indicate<br>the length of AB.   |
| (iv) Arithmetic.         | (iv) A 10.5 meter wire is<br>affixed to a point at the top<br>of a telegraph pole and is<br>anchored in the ground<br>3.6 meters from the pole.<br>Calculate the length of the<br>pole. |

#### 4.2.8 Evaluating and reducing the learning material

a) *Theme*: (Trigonometry, Grades 11 and 12).  
Simple identities that pertain to the relationships mentioned in the  
above section.

b) *Reduction of the theme*

The essence of this theme is: To simplify the left and the right sides  
of the identity to the same expression.

#### Theory

Prove that  $\tan A + \cot A \cong$   
 $\operatorname{Cosec}^2 A \cdot \sec A \cdot \sin A$ .

(i) *Left side*: Simplify to  
a sin- and cos-function.

#### Illustration

(i) Left side

$$\begin{aligned}
&= \tan A + \cot A \\
&= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \\
&= \frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A} \\
&= \frac{1}{\cos A \cdot \sin A}
\end{aligned}$$

(ii) *Right side:* Simplify to a sin- and cos-function.

(ii) Right side

$$\begin{aligned}
&= \operatorname{cosec}^2 A \cdot \sec A \cdot \sin A \\
&= \frac{1}{\sin^2 A} \times \frac{1}{\cos A} \times \sin A \\
&= \frac{1}{\sin A \cdot \cos A}
\end{aligned}$$

(iii) The solution now must necessarily fall open.

$$\therefore \tan A + \cot A = \operatorname{cosec}^2 A \cdot \sec A \cdot \sin A.$$

c) *Evaluating*

The following examples can be given to the pupils on a test:

(i) For the proof of the identity:  $\tan A + \cot A = \operatorname{cosec}^2 A \sec A \sin A$ , the pupils alone must fill in the open spaces of the following exposition.

Left side =  $\tan A + \cot A$ .

$$\begin{aligned}
&= \dots\dots\dots \\
&= \frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A} \\
&= \dots\dots\dots
\end{aligned}$$

Right side =  $\operatorname{cosec}^2 A \cdot \sec A \cdot \sin A$ .

$$\begin{aligned}
&= \dots\dots\dots \\
&= \dots\dots\dots
\end{aligned}$$