

This question is posed, on the one hand, so as to evaluate the pupils' skillfulness and knowledge with applying the newly acquired insights. On the other hand, by this the possibility that the pupils are going to make errors with irrelevant manipulations is eliminated.

(ii) Prove that $\tan^2 x \cong \sin^2 x + \sin^2 x \cdot \tan^2 x$. With this example, on the one hand, there is an attempt to evaluate insight into the learning aim (implementing relationships). On the other hand, the mobility of the child regarding the application of his acquisitions in a new situation is tested.

Theory

(i) Common factor

(ii) Relationship:

$$1 + \tan^2 x = \sec^2 x.$$

(iii) Relationship:

$$\sec^2 x = \frac{1}{\cos^2 x}$$

(iv) Relationship:

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$

(v) Solution.

Illustration

Right side

$$\begin{aligned} &= \sin^2 x + \sin^2 x \tan^2 x \\ \text{(i)} &= \sin^2 x (1 + \tan^2 x) \\ &= \sin^2 x (\sec^2 x) \\ &= \sin^2 x \frac{1}{\cos^2 x} \\ &= \frac{\sin^2 x}{\cos^2 x} \end{aligned}$$

Left side

$$\begin{aligned} &= \tan^2 x \\ &= \frac{\sin^2 x}{\cos^2 x} \end{aligned}$$

$$\therefore \tan^2 x = \sin^2 x + \sin^2 x \cdot \tan^2 x.$$

4.2.9 Summary

Because the various facets of the lesson structure cannot exist meaningfully apart from each other a summary example is given within which the complete course of a lesson is planned in terms of a particular theme. Each step sequentially is brought into harmony with the insights acquired by reducing the learning material to its essences.

4.3 AN EXAMPLE OF A COMPLETE LESSON STRUCTURE

a) *Theme:* (Trigonometry, Grades 11 and 12).

Solving trigonometric equations. Solutions must be limited to the interval $[-360^\circ; 360^\circ]$.

b) *Reduction of the theme*

The essence of this theme is determining the collection of all *sets* of angles within the previously mentioned interval that satisfies the equation. This embraces the reversal of the formula by *simplifying* and/or *factoring* it into a form by which the solution can be found from the relevant tables.

Theory	Illustration
<i>The course of the solution</i>	$2 \cos x = 3 \tan x.$
(i) Substitution.	(i) $2 \cos x = 3 \sin x / \cos x$
(ii) Simplifying.	(ii) $2 \cos^2 x = 3 \sin x.$
(iii) Substitution.	(iii) $2(1 - \sin^2 x) = 3 \sin x.$ $2 - 2 \sin^2 x = 3 \sin x.$ $-2 \sin^2 x - 3 \sin x + 2 = 0$
(iv) General form.	(iv) $2 \sin^2 x + 3 \sin x + 2 = 0$
(v) Factorizing.	(v) $(2 \sin x - 1)(\sin x + 2) = 0$
(vi) Solving for the size of the unknown angles from the tables.	(vi) $2 \sin x - 1 = 0$ $\sin x + 2 = 0$ 0 $2 \sin x = 1$ $\sin x = -2$ $\sin x = 1/2$ unsolvable $x = 30^\circ + k360^\circ$ or $x = 150^\circ + k360^\circ$ $x = 30^\circ; 150^\circ$ $-330^\circ; -210^\circ.$

c) *The learning aim*

The essence of the theme as shown by the reduction activity of the teacher is summarized as follows:

- (i) The effective implementation of known *identities* and *factors* by simplifying the equations to a *general form* by which the solutions can be found from the relevant tables.

- (ii) The solution of the unknown size of the angles within the given interval that the equation satisfies.

d) *Lesson aim*

In compliance with the learning aim now the teacher proceeds to plan his lesson aim. That is, he anticipates a minimum amount of foreknowledge, an effective lesson form and effective modes of learning. The lesson aim only can be realized in the course of the lesson. Therefore, the lesson form and didactic modalities that are planned must be indicated for each phase of the lesson.

Actualizing foreknowledge

The course of a lesson only can have a meaningful beginning if room is made in the lesson aim for actualizing relevant foreknowledge. The following is a guide to what foreknowledge must be actualized in solving trigonometric equations.

(i) *Known identities*

(i)

$$\left. \begin{aligned} \sin x &= \frac{1}{\operatorname{cosec} x} \\ \cos x &= \frac{1}{\sec x} \\ \tan x &= \frac{\sin x}{\cos x} \\ \tan x &= \frac{1}{\cot x} \\ \sin^2 x + \cos^2 x &= 1 \\ \tan^2 x + 1 &= \sec^2 x \\ 1 + \cot^2 x &= \operatorname{cosec}^2 x \end{aligned} \right\}$$

(ii) *Factors*

1. Trinomial.
2. Grouping.
3. Common factor.

(ii)

1. $2x^2 + 3x + 2$.
2. $5xy - 4x - 15y + 12$.
3. Compare 2.

(iii) *Quadratic equations.*

(iii) $2x^2 + 3x - 2 = 0$