

(iv) The use of the *natural tables*.

(iv) From the tables determine the size of x if $\sin x = 0.5$.

The following lesson form and didactic modalities are anticipated for this phase of the lesson.

A. Lesson form

B. Didactic modalities

a) *Didactic ground form(s)*.
Conversation and example.

a) *Principles of actualization*
Guided activity.
Guided individualization.

b) *Methodological principle*.
Mainly inductive.

b) *Modes of learning*.
Remembering.
Reviewing and exercising
Known insights.

c) *Principle of ordering learning material*.
Linear.

c) *Teaching aids*.
Schematizing on the
blackboard.

Stating and formulating the problem

In terms of the relevant foreknowledge actualized the pupils are made aware that at this stage they can solve algebraic equations and even simple “trigonometric equations” such as $\sin x = 0.5$.

However, the first totality view of an equation such as $2 \cos x = 3 \tan x$ now immediately awakens in the child a feeling of not knowing, of strangeness or something problematic. By a closer investigation (perceiving), i.e., by analyzing and “breaking open” the problem (trigonometric equations) certain things still remain unsolvable. The child with his foreknowledge of algebraic equations cannot independently arrive at a solution. With a good example the child now is helped to formulate the problem for himself:

The problem with respect to trigonometric equations can be summarized in the following two steps:

Example

(i) Defining the original equation as an equation with only one function.

(i) *Original equation.*
 $2 \cos x = 3 \tan x.$
Changed equation.
 $2\sin^2x + 3\sin x - 2 = 0.$

(ii) Factoring the new equation.

(ii) $(2\sin x - 1)(\sin x + 2) = 0$

The following lesson form and didactic modalities are anticipated for this phase of the course of the lesson.

A. Lesson form

- a) *Didactic ground forms.*
Conversation and example.
- b) *Methodological principle.*
Inductive and deductive.
- c) *Principle of ordering content.*
Punctual and spiral.

B. Didactic modalities

- a) *Principles of actualization*
 - (i) *Stating problem.*
Guided activity.
 - (ii) *Formulating problem*
Self activity.
- b) *Modes of learning.*
Sensing and perceiving.
- c) *Teaching aid.*
Blackboard.

Exposing the new

The difference as well as the similarity between solving a trigonometric and an algebraic equation can be shown to the pupils with a good example. From the statement of the problem, the following steps therefore are necessary in exposing the new.

New theory

(i) Defining the original equation as an equation with only one unknown.

a) Known identity.

b) Simplifying: Least Common Multiple.

(i) $2 \cos x = 3 \tan x.$

a) Put $\tan x = \frac{\sin x}{\cos x}$
in equation (i).

$$\therefore 2 \cos x = \frac{3 \sin x}{\cos x}$$

b) $2 \cos^2 x = 3 \sin x$

c) Known identity.

d) Simplifying. Remove brackets and bring all terms to one side.

e) Multiply by (-1).

(ii) Factor the *trinomial*.

c) Put $\cos^2x = 1 - \sin^2x$ in (b):

$$2(1 - \sin^2x) = 3\sin x.$$

d) $2 - 2\sin^2x = 3\sin x.$

$$-2\sin^2x = 3\sin x + 2 = 0$$

e) $2\sin^2x + 3\sin x - 2 = 0$

$$(ii) (2\sin x - 1)(\sin x + 2) = 0.$$

Foreknowledge for determining angle sizes,

a) Each term of the equation now can be equated with *zero*.

b) Determine angle sizes from tables.

$$\begin{array}{l|l} a) 2\sin x - 1 = 0 & \sin x + 2 = 0 \\ 2\sin x = 1 & \sin x = -2 \\ \sin x = 1/2 & \text{unsolv.} \end{array}$$

$$\begin{array}{l} x = 30^\circ + k360^\circ \\ \text{or } x = 150^\circ + \\ k360^\circ \\ x = 30^\circ; 150^\circ; \\ -330^\circ; -210^\circ. \end{array}$$

From the above it is clear that the new insights are made meaningful on the basis of the foreknowledge and then are integrated with each other into a new or “different” structure.

The following lesson form and didactic modalities are anticipated for this phase.

A. Lesson form

a) *Didactic ground form(s)*.
Conversation and example.

b) *Methodological principles*.
Inductive and deductive.

c) *Principle of ordering content*.
Punctual, concentric.

B. Didactic modalities

a) *Principles of actualization*
Guided activity.
Guided tempo.

b) *Modes of learning*.
Perceive and think.

c) *Teaching aid*.
Blackboard.