Actualizing the learning content

The following example can be done together with the pupils to implement the learning content: Solve for x if 5 cos x + 12 cosec x = Cot x if x $\in$ [-360^0; 360^0.]

This example is worked through with the pupils to provide security and confidence regarding *simplifying* the equation by applying known identities as well as because through simplification the equation is changed to a form where the factors can be acquired through *grouping*.

The following is given as a concrete illustration of this part-structure.

Example that is worked through with the pupils:

(i) Example

(i) 5 cos x + 12 cosec x = 15 + 4 cot x.

(ii) Substitution
Identity: cosec x =

(ii) $5\cos x + \frac{12}{\sin x} = 15 + \frac{4\cos x}{\sin x}$

$5\cos x\sin x + 12 = 15\sin x + 4\cos x.$

5 cos x sin x – 4 cos x – 15 sin x + 12 = 0.
cos x(5 sin – 4) – (5 sin x – 4) = 0.

(iii) *Factors:* Grouping.

(iii) (5 sin x – 4)(cos x – 3) = 0.

5 sin x – 4 = 0	cos x – 3=0.
5 sin x = 4.	
sin x = 4/5	cos x = 3.
sin x = .8	unsolv.

(iv) Read the size of the angles from the tables.

(iv) $x = 53^0 8' + k360^0$
or $x = 126^0 52' + k360^0$
$x = 53^0 8'$; $126^0 52'$;
$-306^0 52'$; $- 233^0 8'$.

The following lesson form and didactic modalities are anticipated here.

A. Lesson form

a) *Didactic ground forms*
Conversation and example.

b) *Methodological principle*
Deductive.

c) *Principles for ordering contgent*
Punctual, linear.

B. Didactic modalities

a) *Principles of actualization*
Guided and self activity.
Guided tempo.

b) *Modes of learning*
Perceive, think and Practice (imitate).

c) *Teaching aid*
Blackboard.

Functionalizing new insights

The following examples can be worked through by the pupils themselves to practice the newly acquired insights and integrate them with their knowledge on hand.

(i) *First example that must be worked through by each pupil*
Solve for x if $6 \cos^2 x = 10 + 11 \sin x$ if $x \in [-360^0; 360^0.]$ With this example the aim is to *practice simplifying* by applying *identities* and then analyzing the resulting factors. Also, there is a striving for an integration of the new with the already available insights regarding matters such as *removing brackets, solving quadratic equations* and the terrain of compiling values of sin x (namely: $-1 \leq \sin x \leq 1$).

Concrete illustration of the example:

(i) Example.

(i) $6 \cos^2 x = 10 + 11 \sin x$.

(ii) Simplifying: Quadratic identity.

(ii) $6 (1 - \sin^2 x) = 10 + 11 \sin x$.

$6 - 6\sin^2 x = 10 + 11\sin x.$

(iii) Removing brackets.

(iii) $-6\sin^2 x - 11\sin x - 4 = 0.$

(iv) Write in the general form.

(iv) $6\sin^2 x + 11\sin x + 4 = 0.$

(v) Analyze into factors:

(v) $(2\sin x + 1)(3\sin x + 4) = 0.$

(vi) Determine possible angle values.

$2\sin x = -1$	$3\sin x + 4 = 0.$
$\sin x = -1/2$	$3\sin x = -4$
$x = 210^0 + k360^0$	$\sin x = -4/3$
or $x = 330^0 + k360^0$	

(vii) The *terrain* of sin x.

(vi) $x = 210^0; 330^0; -150^0; -30^0.$	unsolv.

(ii) *Second example that must be worked through by each pupil* Solve for x if $3\cos x - 2 = 3 - 2\cot x$ and $\in[-360^0; 360^0.]$ With this example the aim is to *practice simplifying* by applying *known identities* and analyzing into factors by *grouping*. Also, it creates an opportunity for integrating the new with the already existing insight regarding solving equations in a fraction form. The following is an attempt to concretely illustrate this aim.

(i) Second example.

(i) $3\cos x - 2\operatorname{cosec} x = 3 - 2\cot x.$

(ii) Simplifying: Known identities:

(ii) $3\cos x - \frac{2}{\sin x} = 3 - \frac{2\cos x}{\sin x}$